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Phenomenology of S_4 Flavor Symmetric extra U(1) model

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Abstract

We study several phenomenologies of an E_6 inspired extra U(1) model with S_4 flavor symmetry. With the assignment of left-handed quarks and leptons to S_4-doublet, SUSY flavor problem is softened. As the extra Higgs bosons are neutrinophilic, baryon number asymmetry in the universe is realized by leptogenesis without causing gravitino overproduction. We find that the allowed region for the lightest chargino mass is given by 100-140 GeV, if the dark matter is a singlino dominated neutralino whose mass is about 36 GeV.

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1 Introduction

Standard model (SM) is a successful theory of gauge interactions, however there are many unsolved puzzles in the Yukawa sectors. What do the Yukawa hierarchies of quarks and charged leptons mean? Why is the neutrino mass so small? Why does the generation exist? These questions give rise to the serious motivation to extend SM. Another important puzzle of SM is the existence of large hierarchy between electroweak scale $M_W \sim 10^2$GeV and Planck scale $M_P \sim 10^{18}$GeV. The elegant solution of this hierarchy problem is supersymmetry (SUSY)[1]. Recent discovery of the Higgs boson at the Large Hadron Collider (LHC) may suggest the existence of SUSY because the mass of Higgs boson; $125 - 126$ GeV [2], is in good agreement with the SUSY prediction. Moreover, in the supersymmetric model, more information are provided for the Yukawa sectors.

In the supersymmetric model, the Yukawa interactions are introduced in the form of superpotential. Therefore, to understand the structure of the Yukawa interaction, we have to understand the structure of superpotential. In the minimal supersymmetric standard model (MSSM), as the Higgs superfields H^U and H^D are vector-like under the SM gauge symmetry $G_{SM} = SU(3) \times SU(2) \times U(1)$, we can introduce μ-term;

$$\mu H^U H^D, \tag{1}$$

in superpotential. The natural size of parameter μ is $O(M_P)$, however μ must be $O(M_W)$ to succeed in breaking electroweak gauge symmetry. This is so-called μ-problem. The elegant solution of μ-problem is to make Higgs superfields chiral under a new $U(1)_X$ gauge symmetry. Such a model is achieved based on E_6-inspired extra U(1) model [3]. The new gauge symmetry replaces the μ-term by trilinear term;

$$\lambda S H^U H^D, \tag{2}$$

which is converted into effective μ-term when singlet S develops $O(1\text{TeV})$ vacuum expectation value (VEV) [4]. At the same time, the baryon and lepton number violating terms in MSSM are replaced by single G-interactions;

$$GQQ + G^c U^c D^c + G U^c E^c + G^c Q L, \tag{3}$$

where G and G^c are new colored superfields which must be introduced to cancel gauge anomaly. These terms induce very fast proton decay. To make proton stable, we must tune these trilinear coupling constants to be very small $\sim O(10^{-14})$, which gives rise to a new puzzle.

The existence of small parameters in superpotential suggests that a new symmetry is hidden. As such a symmetry suppresses the Yukawa coupling of the first and the second generation of the quarks and the charged leptons, it should be flavor symmetry. We guess several properties that the flavor symmetry should have in order. At first, the flavor symmetry should be non-abelian and include triplet representations, which is the simple reason why three generations exist. At second, remembering that the quark and the charged-lepton masses are suppressed by $SU(2)_W$ gauge symmetry as the left-handed fermions are assigned to be doublet and the right-handed fermions are assigned to be singlet, the flavor symmetry should include doublets. In this case, if we assign the first and the second generation of the left-handed quarks and leptons to be doublets and the right-handed to be singlets, then suppression of Yukawa couplings is realized in the same manner as $SU(2)_W$. At the same time, this assignment softens the SUSY-flavor problem because of the left-handed sfermion mass degeneracy. Finally, any products of the doublets should not include the triplets. In this case, we can forbid single G-interactions when we assign G and G^c to be triplets and the others to be doublets or singlets. As one of the candidates of the flavor symmetries which have the nature as above, we consider S_4 [5]. In such a model, the generation structure is understood as a new system to stabilize proton [6].

In section 2, we introduce new symmetries and explain how to break them. In section 3, we discuss Higgs multiplets. In section 4, we give order-of-magnitude estimates of the mass matrices of quarks and leptons and flavor changing processes. In section 5, we discuss cosmological aspects of our model. Finally, we give conclusions in section 6.

2 Symmetry Breaking

At first we introduce new symmetries and explain how to break these symmetries. The charge assignments of the superfields are also defined in this section.

2.1 Gauge symmetry

We extend the gauge symmetry from G_{SM} to $G_{32111} = G_{SM} \times U(1)_X \times U(1)_Z$, and add new superfields N^c, S, G, G^c which are embedded in **27** representation of E_6 with quark, lepton superfields Q, U^c, D^c, L, E^c and Higgs superfields H^U, H^D. Where N^c is right-handed neutrino (RHN), S is G_{SM} singlet and G, G^c are colored Higgs. The two U(1)s are linear combinations of $U(1)_\psi, U(1)_\chi$ where $E_6 \supset SO(10) \times U(1)_\psi \supset SU(5) \times U(1)_\chi \times U(1)_\psi$, and their charges X and Z are given as follows

$$X = \frac{\sqrt{15}}{4} Q_\psi + \frac{1}{4} Q_\chi, \quad Z = -\frac{1}{4} Q_\psi + \frac{\sqrt{15}}{4} Q_\chi. \tag{4}$$

The charge assignments of the superfields are given in Table 1. To break $U(1)_Z$, we add new vector-like superfields Φ, Φ^c where Φ^c is the same representation as RHN under the G_{32111} and its anti-representation Φ is originated in $\mathbf{27}^*$. To discriminate between N^c and Φ^c, we introduce Z_2^R symmetry and assign Φ^c, Φ to be odd. The invariant superpotential under these symmetries is given by

$$\begin{aligned}
W_{32111} &= W_0 + W_S + W_G + W_\Phi, && (5)\\
W_0 &= Y^U H^U Q U^c + Y^D H^D Q D^c + Y^L H^D L E^c + Y^N H^U L N^c + \frac{Y^M}{M_P} \Phi\Phi N^c N^c, && (6)\\
W_S &= kSGG^c + \lambda S H^U H^D, && (7)\\
W_G &= Y^{QQ} GQQ + Y^{UD} G^c U^c D^c + Y^{UE} G U^c E^c + Y^{QL} G^c Q L + Y^{DN} G D^c N^c, && (8)\\
W_\Phi &= M_\Phi \Phi\Phi^c + \frac{1}{M_P} Y^\Phi (\Phi\Phi^c)^2, && (9)
\end{aligned}$$

where unimportant higher dimensional terms are omitted. Since the interactions W_S drive squared mass of S to be negative through renormalization group equations (RGEs), spontaneous $U(1)_X$ symmetry breaking is realized and $U(1)_X$ gauge boson Z' acquires the mass

$$m(Z') = 5\sqrt{2} g_x \langle S \rangle = 5\sqrt{2} \left(\frac{1}{2\sqrt{6}} g_X \right) \langle S \rangle = 0.5255 \langle S \rangle, \tag{10}$$

where the used value $g_X(M_S = 1\text{TeV}) = 0.3641$ is calculated based on the RGEs given in appendix A, and $\langle H^{U,D} \rangle \ll \langle S \rangle$ is assumed based on the experimental constraint

$$m(Z') > 1.52\text{TeV}[7], \tag{11}$$

which imposes lower bound on VEV of S as

$$\langle S \rangle > 2892\text{GeV}. \tag{12}$$

To drive squared mass of Φ^c to be negative, we introduce 4th generation superfields H_4^U, L_4 and their anti-representations $\bar{H}_4^U, \bar{L}_4$ and add new interaction

$$W \supset Y^{LH} \Phi^c H_4^U L_4. \tag{13}$$

To forbid the mixing between 4th generation and three generations, we introduce 4-th generation parity $Z_2^{(4)}$ and assign all 4-th generation superfields to be odd. If $M_\Phi = 0$ in W_Φ, then Φ, Φ^c develop large VEVs along the D-flat direction of $\langle \Phi \rangle = \langle \Phi^c \rangle = V$, $U(1)_Z$ is broken and $U(1)_Z$ gauge boson Z'' acquires the mass

$$m(Z'') = 8 g_z V = 8 \left(\frac{1}{6} \sqrt{\frac{5}{2}} g_Z \right) V = 0.9202V, \tag{14}$$

where the used value $g_Z(\mu = M_I) = 0.4365$ is calculated by the same way as g_X. We determine the values of two gauge couplings g_X, g_Z by requiring three $U(1)$ gauge coupling constants are unified at reduced Planck scale $M_P = 2.4 \times 10^{18}$GeV as

$$g_Y(M_P) = g_X(M_P) = g_Z(M_P). \tag{15}$$

In this paper we fix the VEV as

$$V = M_I = 10^{11.5}\text{GeV}. \tag{16}$$

RHN obtains the mass

$$M_R \sim \frac{V^2}{M_P} \sim 10^{4-5}\mathrm{GeV}, \tag{17}$$

through the quartic term in W_0.

After the gauge symmetry breaking, since the R-parity symmetry defined by

$$R = Z_2^R \exp\left[\frac{i\pi}{20}(3x - 8y + 15z)\right], \tag{18}$$

remains unbroken, the lightest SUSY particle (LSP) is a promising candidate for cold dark matter. As we adopt the naming rule of superfields as the name of superfield is given by its R-parity even component, we call G, G^c "colored Higgs".

Before considering flavor symmetry, we should keep in mind following points. As the interaction W_G induces too fast proton decay, they must be strongly suppressed. The mass parameter M_Φ in W_Φ must be forbidden in order to break $U(1)_Z$ symmetry. In W_0, the contributions to flavor changing processes from the extra Higgs bosons must be suppressed [8].

	Q	U^c	E^c	D^c	L	N^c	H^D	G^c	H^U	G	S	Φ	Φ^c
$SU(3)_c$	3	$\bar{3}$	1	$\bar{3}$	1	1	1	$\bar{3}$	1	3	1	1	1
$SU(2)_w$	2	1	1	1	2	1	2	1	2	1	1	1	1
$y = 6Y$	1	-4	6	2	-3	0	-3	2	3	-2	0	0	0
$6\sqrt{2/5}Q_\psi$	1	1	1	1	1	1	-2	-2	-2	-2	4	-1	1
$2\sqrt{6}Q_\chi$	-1	-1	-1	3	3	-5	-2	-2	2	2	0	5	-5
$x = 2\sqrt{6}X$	1	1	1	2	2	0	-3	-3	-2	-2	5	0	0
$z = 6\sqrt{2/5}Z$	-1	-1	-1	2	2	-4	-1	-1	2	2	-1	4	-4
Z_2^R	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$-$
R	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

Table 1: G_{32111} assignment of superfields. Where the x, y and z are charges of $U(1)_X$, $U(1)_Y$ and $U(1)_Z$, and y is hypercharge. The charges of $U(1)_\psi$ and $U(1)_\chi$ which are defined in Eq.(4) are also given.

2.2 S_4 flavor symmetry

If we introduce S_4 flavor symmetry and assign G, G^c to be triplets, then W_G defined in Eq.(8) is forbidden. This is because any products of doublets and singlets of S_4 do not contain triplets. The multiplication rules of representations of S_4 are given in appendix B. Note that we assume full E_6 symmetry does not realize at Planck scale, therefore there is no need to assign all superfields to the same flavor representations. In this model the generation number three is imprinted in G, G^c. Therefore they may be called "G-Higgs" (generation number imprinted colored Higgs).

Since the existence of G-Higgs which has life time longer than 0.1 second spoils the success of Big Ban nucleosynthesis (BBN)[9], S_4 symmetry must be broken. Therefore we assign Φ to be triplet and Φ^c to be doublet and singlet to forbid $M_\Phi \Phi\Phi^c$. With this assignment, S_4 symmetry is broken due to the VEV of Φ and the effective trilinear terms are induced by pentatic terms

$$W_{NRG} = \frac{1}{M_P^2}\Phi\Phi^c\left(GQQ + G^cU^cD^c + GE^cU^c + G^cLQ + GD^cN^c\right). \tag{19}$$

The size of effective coupling constants of these terms is given by

$$\frac{\langle\Phi\rangle\langle\Phi^c\rangle}{M_P^2} \sim \frac{M_R}{M_P} \sim 10^{-14}. \tag{20}$$

This is the marginal size to satisfy the BBN constraint [10]. This relation gives the information about the RHN mass scale if the life time of G-Higgs is measured.

The assignments of the other superfields are determined based on following criterion, (1)The quark and charged lepton mass matrices reproduce observed mass hierarchies and CKM and MNS matrices. (2)The third

generation Higgs H_3^U, H_3^D are specified as MSSM Higgs and the first and second generation Higgs superfields $H_{1,2}^U$ are neutrinophilic which are needed for successful leptogenesis.

To realize Yukawa hierarchies, we introduce gauge singlet and S_4 doublet flavon superfield D_i and fix the VEV of D_i by

$$V_D = \sqrt{|\langle D_1\rangle|^2 + |\langle D_2\rangle|^2} = 0.1 M_P = 2.4\times 10^{17}\mathrm{GeV}, \tag{21}$$

then the Yukawa coupling constants are expressed in the power of the parameter

$$\epsilon = \frac{V_D}{M_P} = 0.1, \tag{22}$$

which is realized by Z_{17} symmetry. To drive the squared mass of flavon to be negative, we add 5-th and 6-th generation superfields $L_{5,6}, D_{5,6}^c$ as S_4-doublets and their anti-representations $\bar{L}_{5,6}, \bar{D}_{5,6}^c$ and introduce trilinear terms as

$$\begin{aligned} W_5 &= Y^{DD}[D_1(D_5^c\bar{D}_6^c + D_6^c\bar{D}_5^c) + D_2(D_5^c\bar{D}_5^c - D_6^c\bar{D}_6^c)] \\ &+ Y^{LL}[D_1(L_5\bar{L}_6 + L_6\bar{L}_5) + D_2(L_5\bar{L}_5 - L_6\bar{L}_6)], \end{aligned} \tag{23}$$

where the mass scale of these fields is given by

$$M_{L_5} = Y^{DD}V_D = Y^{LL}V_D = \epsilon M_P = 2.4\times 10^{17}\mathrm{GeV}. \tag{24}$$

We assign the 5th and 6-th generation superfields to be $Z_2^{(5)}$-odd. The representation of all superfields under the flavor symmetry is given in Table 2. The mass terms of 4-th generation fields are given by

$$W_4 = Y^{LH}\Phi_3^c L_4 H_4^U + Y^L\frac{(D_1^2+D_2^2)^2}{M_P^3}L_4\bar{L}_4 + Y^H\frac{(D_1^2+D_2^2)^2}{M_P^3}H_4^U\bar{H}_4^U, \tag{25}$$

where

$$M_{L_4} = \epsilon^4 Y^L M_P = \epsilon^4 Y^H M_P = 2.2\times 10^{14}\mathrm{GeV}, \tag{26}$$

which realizes gauge coupling unification at Planck scale as

$$g_3(M_P) = g_2(M_P). \tag{27}$$

2.3 SUSY breaking

For the successful leptogenesis, the symmetry $Z_2^{(2)}\times Z_2^N$ must be broken softly. Therefore we assume these symmetries are broken in hidden sector and the effects are mediated to observable sectors by gravity. We introduce hidden sector superfields $A, B_+, B_{1,-}, B_{2-}, C_+, C_{1,-}, C_{2-}$, where their representations are given in Table 3.

We construct O'Raifeartaigh model by these hidden sector superfields as follow [11]

$$\begin{aligned} W_{\text{hidden}} &= -M^2A + m_+B_+C_+ + m_{1-}B_{1-}C_{1-} + m_{2-}B_{2-}C_{2-} \\ &+ \frac{1}{2}\lambda_+AC_+^2 + \frac{1}{2}\lambda_{1-}AC_{1-}^2 + \frac{1}{2}\lambda_{2-}AC_{2-}^2. \end{aligned} \tag{28}$$

As the F-terms of hidden sector superfields given by

$$F_A = -M^2 + \frac{1}{2}\lambda_+C_+^2 + \frac{1}{2}\lambda_{1-}C_{1-}^2 + \frac{1}{2}\lambda_{2-}C_{2-}^2, \tag{29}$$

$$F_{B_+} = m_+C_+, \tag{30}$$

$$F_{B_{1-}} = m_{1-}C_{1-}, \tag{31}$$

$$F_{B_{2-}} = m_{2-}C_{2-}, \tag{32}$$

$$F_{C_+} = m_+B_+ + \lambda_+AC_+, \tag{33}$$

$$F_{C_{1-}} = m_-B_{1-} + \lambda_{1-}AC_{1-}, \tag{34}$$

$$F_{C_{2-}} = m_-B_{2-} + \lambda_{2-}AC_{2-}, \tag{35}$$

	Q_i	Q_3	U_1^c	U_2^c	U_3^c	D_1^c	D_2^c	D_3^c	L_i
S_4	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{2}$
$Z_2^{(2)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Z_2^N	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Z_{17}	$2/17$	0	$4/17$	$1/17$	0	$3/17$	$2/17$	$2/17$	$2/17$
Z_2^R	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(4)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(5)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

	L_3	E_1^c	E_2^c	E_3^c	N_1^c	N_2^c	N_3^c	H_i^U	H_3^U
S_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$Z_2^{(2)}$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$
Z_2^N	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$
Z_{17}	$2/17$	$3/17$	$1/17$	0	0	0	0	$1/17$	0
Z_2^R	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(4)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(5)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

	H_i^D	H_3^D	S_i	S_3	G_a	G_a^c	Φ_a	Φ_3^c	Φ_i^c
S_4	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$
$Z_2^{(2)}$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$+$
Z_2^N	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Z_{17}	$1/17$	0	$16/17$	0	0	0	0	0	0
Z_2^R	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$
$Z_2^{(4)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(5)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

	L_4	$\bar{L}_4$	H_4^U	$\bar{H}_4^U$	D_i	L_J	D_J^c	$\bar{L}_J$	$\bar{D}_J^c$
S_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$
$Z_2^{(2)}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Z_2^N	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
Z_{17}	0	$4/17$	0	$4/17$	$16/17$	0	0	$1/17$	$1/17$
Z_2^R	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(4)}$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
$Z_2^{(5)}$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$

Table 2: $S_4 \times Z_2^{(2)} \times Z_2^N \times Z_{17} \times Z_2^R \times Z_2^{(4)} \times Z_2^{(5)}$ assignment of superfields (Where the indices i and J of the S_4 doublets runs $i = 1, 2$ and $J = 5, 6$ respectively, and the index a of the S_4 triplets runs $a = 1, 2, 3$.)

	A	B_+	B_{1-}	B_{2-}	C_+	C_{1-}	C_{2-}
$Z_2^{(2)}$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
Z_2^N	$+$	$+$	$+$	$-$	$+$	$+$	$-$
Z_2^H	$+$	$-$	$-$	$-$	$-$	$-$	$-$
$U(1)_R$	2	2	2	2	0	0	0

Table 3: $Z_2^{(2)} \times Z_2^N \times Z_2^H \times U(1)_R$ assignment of hidden sector superfields. All these superfields are trivial under the gauge symmetry G_{32111} and flavor symmetry $S_4 \times Z_{17} \times Z_2^R \times Z_2^{(4)} \times Z_2^{(5)}$. The observable sector superfields are Z_2^H-even.

do not have the solution as

$$F_A = F_{B_+} = F_{B_{1-}} = F_{B_{2-}} = 0, \tag{36}$$

supersymmetry is spontaneously broken. The flavor symmetry $Z_2^{(2)} \times Z_2^N$ is also broken.

Since we assume the $U(1)_R$ symmetry is explicitly broken in the higher dimensional terms [12], soft SUSY breaking terms are induced by the interaction terms between observable sector and hidden sector as

$$\mathcal{L}_{SB} = \left\{ \left[\frac{A}{M_P} W^\alpha W_\alpha + \frac{A}{M_P}(c_{ABC} X_A X_B X_C + \cdots) \right]_F + h.c. \right\} + \left[\frac{A^* A}{M_P^2} c_{ab} X_a^* X_b + h.c. \right]_D , \tag{37}$$

where the indices A, B, C runs the species of superfields and the indices a, b, c runs generation numbers. Generally, as the coefficient matrices c_{ab} are not unit matrices, large flavor changing processes are induced by the sfermion exchange. The explicit $Z_2^{(2)} \times Z_2^N$ breaking terms are given by

$$\begin{aligned} \mathcal{L}_{Z_2^{(2)} B} &= \left[\frac{B_+^* B_{1-}}{M_P^2} \frac{(D_1 X_1 + D_2 X_2)^* X_3}{M_P} + h.c. \right]_D \\ &= \epsilon m_{BX}^2 \left(c(X_1)^* X_3 + s(X_2)^* X_3 \right) + h.c. \quad (X = H^U, H^D, S), \end{aligned} \tag{38}$$

$$\mathcal{L}_{Z_2^N B} = \left[\frac{B_+^* B_{2-}}{M_P^2} (c_2 N_2^c + c_3 N_3^c)^* (N_1^c) + h.c. \right]_D = m_{12}^2 (N_2^c)^* N_1^c + m_{13}^2 (N_3^c)^* N_1^c + h.c.. \tag{39}$$

2.4 S_3 breaking

The S_3 subgroup of S_4 is broken by the VEV of S_4-doublet flavon D_i. Here we consider the direction of VEV. For the later convenience, we define the products of D_i as follows,

$$\mathbf{1} \; : \; E_2 = D_1^2 + D_2^2, \quad E_3 = 3D_1^2 D_2 - D_2^3, \tag{40}$$

$$\mathbf{1'} \; : \; P_3 = D_1^3 - 3D_1 D_2^2, \tag{41}$$

$$\mathbf{2} \; : \; V_1 = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 2D_1 D_2 \\ D_1^2 - D_2^2 \end{pmatrix}, \quad V_4 = \begin{pmatrix} -D_2 P_3 \\ D_1 P_3 \end{pmatrix}, \quad V_5 = \begin{pmatrix} -(D_1^2 - D_2^2) P_3 \\ 2D_1 D_2 P_3 \end{pmatrix}, \tag{42}$$

and the VEVs of each components of D_i as

$$\langle D_1 \rangle = V_D c = V_D \cos\theta, \quad \langle D_1 \rangle = V_D s = V_D \sin\theta. \tag{43}$$

Generally, the superpotential of D_i is written in the form of polynomial in E_2, E_3, P_3 as

$$M_P^{14} W_D = a_1 E_2^7 E_3 + a_2 E_2^4 E_3^3 + a_3 E_2^4 P_3^2 E_3 + a_4 E_2 E_3^5 + a_5 E_2 P_3^2 E_3^3 + a_6 E_2 P_3^4 E_3. \tag{44}$$

Substituting the VEVs given in Eq.(43) to the flavon potential, we get

$$V(V_D, \theta) = \left\{ -A[a_1' s_3 + a_2' s_3^3 + a_3' c_3^2 s_3 + a_4' s_3^5 + a_5' c_3^2 s_3^3 + a_6' c_3^4 s_3] V_D^3 \left(\frac{V_D}{M_P} \right)^{14} + h.c. \right\} + V_F + m^2 V_D^2, \tag{45}$$

where

$$s_3 = \sin 3\theta, \quad c_3 = \cos 3\theta, \tag{46}$$

and V_F is F-term contribution. As this potential is polynomial in s_3, the stationary condition

$$\frac{\partial V(V_D, \theta)}{\partial \theta} = c_3 \left[a_0'' + a_1'' s_3 + a_2'' s_3^2 + a_3'' s_3^3 + a_4'' s_3^4 + a_5'' s_3^5 + a_7'' s_3^7 + a_9'' s_3^9 \right] = 0, \tag{47}$$

gives parameter independent solution

$$c_3 = 0, \tag{48}$$

and parameter dependent solution

$$a_0'' + a_1'' s_3 + a_2'' s_3^2 + a_3'' s_3^3 + a_4'' s_3^4 + a_5'' s_3^5 + a_7'' s_3^7 + a_9'' s_3^9 = 0. \tag{49}$$

Which solution of two is selected for the global minimum is depends on the parameters in potential. Since the solution Eq.(48) gives wrong prediction such as massless up-quark and electron, we assume the solution Eq.(49) corresponds to the global minimum. In this paper we assume $\langle D_i \rangle$ are real without any reason, which is important in considering CP violation in section 4.

The scale of V_D is determined by the minimum condition

$$\frac{1}{V_D}\frac{\partial V(V_D,\theta)}{\partial V_D} \sim m^2 + \frac{V_D^{30}}{M_P^{28}} = 0, \tag{50}$$

as

$$\frac{V_D}{M_P} \sim \left(\frac{|m|}{M_P}\right)^{1/15} \sim \left(\frac{10^3\mathrm{GeV}}{10^{18}\mathrm{GeV}}\right)^{1/15} \sim 10^{-1}, \tag{51}$$

which agrees with Eq.(22). In this paper we sometimes write SUSY breaking scalar squared mass parameters as m^2 for simplicity and assume $m \sim O(\mathrm{TeV})$.

2.5 S_4 breaking

The superpotential of gauge non-singlet flavons Φ, Φ^c is given by

$$\begin{aligned} W_\Phi &= \frac{Y_1^\Phi}{M_P}(\Phi_3^c)^2[\Phi_1^2+\Phi_2^2+\Phi_3^2] + \frac{Y_2^\Phi}{M_P}[(\Phi_1^c)^2+(\Phi_2^c)^2][\Phi_1^2+\Phi_2^2+\Phi_3^2] \\ &+ \frac{Y_3^\Phi}{M_P}[2\sqrt{3}\Phi_1^c\Phi_2^c(\Phi_2^2-\Phi_3^2)+((\Phi_1^c)^2-(\Phi_2^c)^2)(\Phi_2^2+\Phi_3^2-2\Phi_1^2)] \\ &+ \frac{Y_4^\Phi}{M_P}\Phi_3^c[\sqrt{3}\Phi_1^c(\Phi_2^2-\Phi_3^2)+\Phi_2^c(\Phi_2^2+\Phi_3^2-2\Phi_1^2)]. \end{aligned} \tag{52}$$

Since the first term in Eq.(25) drives the squared mass of Φ_3^c to be negative through RGEs, these flavons develop VEVs along the D-flat direction as follows

$$\langle\Phi_1^c\rangle = \langle\Phi_2^c\rangle = 0, \quad \langle\Phi_1\rangle = \langle\Phi_2\rangle = \langle\Phi_3\rangle = \frac{\langle\Phi_3^c\rangle}{\sqrt{3}} = \frac{V}{\sqrt{3}}, \tag{53}$$

where S_3-symmetry is unbroken in this vacuum. The scale of V is determined by the minimum condition

$$\frac{1}{V}\frac{\partial V(\Phi)}{\partial \Phi} \sim m^2 + |Y^\Phi|^2\frac{V^4}{M_P^2} = 0, \tag{54}$$

as

$$\frac{V}{M_P} \sim \sqrt{\frac{|m|}{|Y^\Phi|M_P}} \sim \sqrt{\frac{10^3\mathrm{GeV}}{(0.1)10^{18}\mathrm{GeV}}} \sim 10^{-7}, \tag{55}$$

which agrees with Eq.(16). In this paper we define the size of $O(1)$ coefficient as $0.1 < Y^X < 1.0$.

Note that there are S_3 breaking corrections in the potential of Φ, Φ^c as follows

$$\begin{aligned} V(\Phi) &\supset m_1^2\left(\frac{(D_1\Phi_2^c+D_2\Phi_1^c)^*(D_1\Phi_3^c)+(D_1\Phi_1^c-D_2\Phi_2^c)^*(D_2\Phi_3)}{M_P^2}+h.c.\right) \\ &+ m_2^2\left(\frac{|2D_2\Phi_1|^2+|(\sqrt{3}D_1+D_2)\Phi_2|^2+|(\sqrt{3}D_1-D_2)\Phi_3|^2}{M_P^2}\right)+\cdots \\ &= \epsilon^2 m_1^2[s_2(\Phi_1^c)^*\Phi_3^c + c_2(\Phi_2^c)^*\Phi_3^c + h.c.] \\ &+ \epsilon^2 m_2^2[4s^2|\Phi_1|^2+(\sqrt{3}c+s)^2|\Phi_2|^2+(\sqrt{3}c-s)^2|\Phi_3|^2]+\cdots, \end{aligned} \tag{56}$$

the direction given in Eq.(53) is modified as follows

$$\langle\Phi_1^c\rangle \sim \langle\Phi_2^c\rangle \sim O(\epsilon^2)V, \quad \langle\Phi_3^c\rangle = (1+O(\epsilon^2))V, \quad \langle\Phi_a\rangle = (1+O(\epsilon^2))\frac{V}{\sqrt{3}}. \tag{57}$$

3 Higgs Sector

Based on the set up given in section 2, we discuss about phenomenology of our model. In this section, we consider Higgs doublet multiplets H_a^U, H_a^D and singlet multiplets S_a.

3.1 Higgs sector

The superpotential of Higgs sector is given by

$$\begin{aligned}
W_S &= \frac{(\lambda_1)_0}{M_P^2} S_3(D_1^2 + D_2^2)(H_1^U H_1^D + H_2^U H_2^D) \\
&+ \frac{\lambda_1'}{M_P^2} S_3(D_1 H_1^U + D_2 H_2^U)(D_1 H_1^D + D_2 H_2^D) \\
&+ \frac{\lambda_1''}{M_P^2} S_3(D_2 H_1^U - D_1 H_2^U)(D_2 H_1^D - D_1 H_2^D) \\
&+ \frac{\lambda_1'''}{M_P^2} S_3[(2D_1 D_2)(H_1^U H_2^D + H_2^U H_1^D) + (D_1^2 - D_2^2)(H_1^U H_1^D - H_2^U H_2^D)] \\
&+ \frac{\lambda_1''''}{M_P^2} S_3[(D_1 H_2^U + D_2 H_1^U)(D_1 H_2^D + D_2 H_1^D) + (D_1 H_1^U - D_2 H_2^U)(D_1 H_1^D - D_2 H_2^D)] \\
&+ \lambda_3 S_3 H_3^U H_3^D + \lambda_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 (S_1 H_1^U + S_2 H_2^U) H_3^D \\
&+ k S_3 (G_1 G_1^c + G_2 G_2^c + G_3 G_3^c).
\end{aligned} \tag{58}$$

For simplicity we assume

$$\lambda_1' = \lambda_1'' = \lambda_1''' = \lambda_1'''' = 0, \quad (\lambda_1)_0 \epsilon^2 = \lambda_1. \tag{59}$$

The coupling k and λ_3 drive the squared mass of S_3 to be negative.

Omitting $O(\epsilon)$-terms, Higgs potential is given by

$$\begin{aligned}
V &= m_{H^U}^2(|H_1^U|^2 + |H_2^U|^2) + m_{H_3^U}^2 |H_3^U|^2 + m_{H^D}^2(|H_1^D|^2 + |H_2^D|^2) + m_{H_3^D}^2 |H_3^D|^2 \\
&+ m_S^2(|S_1|^2 + |S_2|^2) + m_{S_3}^2 |S_3|^2 \\
&- \left\{\lambda_3 A_3 S_3 H_3^U H_3^D + \lambda_4 A_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 A_5 (S_1 H_1^U + S_2 H_2^U) H_3^D + h.c.\right\} \\
&+ \left|\lambda_3 H_3^U H_3^D\right|^2 + \left|\lambda_4 H_3^U H_1^D + \lambda_5 H_3^D H_1^U\right|^2 + \left|\lambda_4 H_3^U H_2^D + \lambda_5 H_3^D H_2^U\right|^2 \\
&+ \left|\lambda_3 S_3 H_3^D + \lambda_4 (S_1 H_1^D + S_2 H_2^D)\right|^2 + \left|\lambda_5 H_3^D S_1\right|^2 + \left|\lambda_5 H_3^D S_2\right|^2 \\
&+ \left|\lambda_3 S_3 H_3^U + \lambda_5 (S_1 H_1^U + S_2 H_2^U)\right|^2 + \left|\lambda_4 H_3^U S_1\right|^2 + \left|\lambda_4 H_3^U S_2\right|^2 \\
&+ \frac{1}{8} g_2^2 \sum_{A=1}^{3} \left[(H_a^U)^\dagger \sigma_A H_a^U + (H_a^D)^\dagger \sigma_A H_a^D\right]^2 + \frac{1}{8} g_Y^2 \left[|H_a^U|^2 - |H_a^D|^2\right]^2 \\
&+ \frac{1}{2} g_x^2 \left[x_u |H_a^U|^2 + x_d |H_a^D|^2 + x_s |S_a|^2\right]^2 + V_{\text{1-loop}},
\end{aligned} \tag{60}$$

where $V_{\text{1-loop}}$ is 1-loop corrections from Q_3, U_3^c, G_a, G_a^c. The VEVs of H_3^U, H_3^D, S_3 trigger off gauge symmetry breaking at low energy scale. $Z_2^{(2)}$-breaking terms

$$V_{FB} = \epsilon m_{BU}^2 (H_1^U c + H_2^U s)^* H_3^U + \epsilon m_{BU}^2 (H_1^D c + H_2^D s)^* H_3^D + \epsilon m_{BS}^2 (S_1 c + S_2 s)^* S_3 + h.c., \tag{61}$$

enforce S_4-doublets developing VEVs as follows

$$\langle X_1 \rangle \sim c \left(\frac{\epsilon m_{BX}^2}{m_X^2}\right) \langle X_3 \rangle, \quad \langle X_2 \rangle \sim s \left(\frac{\epsilon m_{BX}^2}{m_X^2}\right) \langle X_3 \rangle, \quad X = H^U, H^D, S. \tag{62}$$

Due to the $Z_2^{(2)}$ symmetry, the Yukawa couplings between H_3^U and N^c are forbidden and neutrino Dirac mass is not induced. To give neutrino Dirac mass, we assume the size of VEV of H_i^U is given by

$$\left\langle H_{1,2}^U \right\rangle \sim \epsilon^2 \left\langle H_3^U \right\rangle \sim 1\text{GeV}, \tag{63}$$

and put the $Z_2^{(2)}$ breaking parameters as follows

$$m_{BU}^2 \sim m_{BD}^2 \sim m_{BS}^2 \sim \epsilon m_{\rm SUSY}^2, \tag{64}$$

by hand. The suppression factor $O(\epsilon)$ may be induced by the running based on RGEs, because off diagonal elements of scalar squared mass matrix do not receive the contributions from gaugino mass parameters which tend to make scalar squared mass larger at low energy scale.

We use the notation of VEVs as follows

$$\langle H_i^U \rangle = (c,s)v_u, \quad \langle H_3^U \rangle = v_u', \quad \langle H_i^D \rangle = (c,s)v_d, \quad \langle H_3^D \rangle = v_d', \quad \langle S_i \rangle = (c,s)v_s, \quad \langle S_3 \rangle = v_s', \tag{65}$$

where we fix the values by

$$v_u' = 150.7, \quad v_d' = 87.0, \quad v_s' = 4000, \quad v_{EW} = \sqrt{(v_u')^2 + (v_d')^2} = 174 \quad ({\rm GeV}), \quad \tan\beta = \frac{v_u'}{v_d'} = \tan\frac{\pi}{3}. \tag{66}$$

In this paper, we neglect the contributions from $v_{u,d,s}$ except for neutrino sector. With this approximation, the potential minimum conditions are given as follows,

$$\begin{aligned}
0 = \frac{1}{v_u'}\frac{\partial V}{\partial H_3^U} &= m_{H_3^U}^2 - \lambda_3 A_3 v_s'(v_d'/v_u') + \lambda_3^2 (v_d')^2 + \lambda_3^2 (v_s')^2 + \frac{1}{4}(g_Y^2 + g_2^2)[(v_u')^2 - (v_d')^2] \\
&- 2g_x^2[-2(v_u')^2 - 3(v_d')^2 + 5(v_s')^2] + \frac{1}{2v_u'}\frac{\partial V_{1-{\rm loop}}}{\partial v_u'}, \qquad (67) \\
0 = \frac{1}{v_d'}\frac{\partial V}{\partial H_3^D} &= m_{H_3^D}^2 - \lambda_3 A_3 v_s'(v_u'/v_d') + \lambda_3^2 (v_u')^2 + \lambda_3^2 (v_s')^2 - \frac{1}{4}(g_Y^2 + g_2^2)[(v_u')^2 - (v_d')^2] \\
&- 3g_x^2[-2(v_u')^2 - 3(v_d')^2 + 5(v_s')^2] + \frac{1}{2v_d'}\frac{\partial V_{1-{\rm loop}}}{\partial v_d'}, \qquad (68) \\
0 = \frac{1}{v_s'}\frac{\partial V}{\partial S_3} &= m_{S_3}^2 - \lambda_3 A_3 v_u'(v_d'/v_s') + \lambda_3^2 (v_u')^2 + \lambda_3^2 (v_d')^2 + 5g_x^2[x_u (v_u')^2 + x_d (v_d')^2 + x_s (v_s')^2], \qquad (69)
\end{aligned}$$

where the 1-loop contribution is neglected in Eq.(69), which is unimportant. These equations give the boundary conditions for $m_{H_3^U}^2, m_{H_3^D}^2, m_{S_3}^2$ at SUSY breaking scale $M_S = 10^3$GeV in solving RGEs.

The mass matrices of heavy Higgs bosons are given as follows

$$\begin{aligned}
M_3^2({\rm CP\ even}) &\simeq \begin{pmatrix} \lambda_3 A_3 v_s' v_d'/v_u' & -\lambda_3 A_3 v_s' & 0 \\ -\lambda_3 A_3 v_s' & \lambda_3 A_3 v_s' v_u'/v_d' & 0 \\ 0 & 0 & 50 g_x^2 (v_s')^2 \end{pmatrix} \qquad (70) \\
M_i^2({\rm CP\ even}) &= M_i^2({\rm CP\ odd}) \simeq {\rm diag}\left(m_{H^U}^2 - 10g_x^2(v_s')^2, m_{H^D}^2 - 15g_x^2(v_s')^2, m_S^2 + 25g_x^2(v_s')^2\right) \qquad (71) \\
M_3^2({\rm CP\text{-}odd}) &\simeq \lambda_3 A_3 v_s' \begin{pmatrix} v_d'/v_u' & 1 & 0 \\ 1 & v_u'/v_d' & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (72) \\
M_3^2({\rm charged}) &\simeq \lambda_3 A_3 v_s' \begin{pmatrix} v_d'/v_u' & 1 \\ 1 & v_u'/v_d' \end{pmatrix} \qquad (73) \\
M_i^2({\rm charged}) &\simeq {\rm diag}\left(m_{H^U}^2 - 10g_x^2(v_s')^2, m_{H^D}^2 - 15g_x^2(v_s')^2\right), \qquad (74)
\end{aligned}$$

where only $O(1{\rm TeV})$ terms are considered and notation $H^U H^D = (H^U)^0(H^D)^0 - (H^U)^+(H^D)^-$ is used. In this approximation, generation mixing terms are negligible. The third generation mass matrices are diagonalized as follows

$$\begin{aligned}
M_3^2({\rm CP\ even}) &= {\rm diag}\left(0, \frac{2\lambda_3 A_3 v_s'}{\sin 2\beta}, 50g_x^2(v_s')^2\right), \qquad (75) \\
M_3^2({\rm CP\ odd}) &= {\rm diag}\left(0, \frac{2\lambda_3 A_3 v_s'}{\sin 2\beta}, 0\right), \qquad (76) \\
M_3^2({\rm charged}) &= {\rm diag}\left(0, \frac{2\lambda_3 A_3 v_s'}{\sin 2\beta}\right), \qquad (77)
\end{aligned}$$

where the zero eigenvalue in CP even Higgs bosons corresponds to lightest neutral CP even Higgs boson and the other zero eigenvalues are Nambu-Goldstone modes absorbed into gauge bosons.

3.2 Lightest neutral CP even Higgs boson

To calculate the mass of the lightest neutral CP even Higgs boson, we diagonalize 2×2 sub-matrix given by

$$
\begin{aligned}
M_3^2(\text{even}) &= \begin{pmatrix} m_{uu,3}^2 & m_{ud,3}^2 \\ m_{ud,3}^2 & m_{dd,3}^2 \end{pmatrix}, && (78) \\
m_{uu,3}^2 &= \lambda_3 A_3(v_s' v_d'/v_u') + \frac{1}{2}(g_Y^2+g_2^2)(v_u')^2 + 8g_x^2(v_u')^2 + \frac{1}{2}\frac{\partial^2 V_{1-\text{loop}}}{\partial (v_u')^2} - \frac{1}{2v_u'}\frac{\partial V_{1-\text{loop}}}{\partial v_u'}, && (79) \\
m_{ud,3}^2 &= -\lambda_3 A_3 v_s' + 2\lambda_3^2 v_u' v_d' - \frac{1}{2}(g_Y^2+g_2^2)v_u' v_d' + 12 g_x^2 v_u' v_d' + \frac{1}{2}\frac{\partial^2 V_{1-\text{loop}}}{\partial v_u' \partial v_d'}, && (80) \\
m_{dd,3}^2 &= \lambda_3 A_3(v_s' v_u'/v_d') + \frac{1}{2}(g_Y^2+g_2^2)(v_d')^2 + 18g_x^2(v_d')^2 + \frac{1}{2}\frac{\partial^2 V_{1-\text{loop}}}{\partial (v_d')^2} - \frac{1}{2v_d'}\frac{\partial V_{1-\text{loop}}}{\partial v_d'}, && (81)
\end{aligned}
$$

where $O(v_{EW})$ terms are included. We evaluate the 1-loop contributions from top, stop, G-Higgs and G-higgsino as

$$
V_{1-\text{loop}} = \frac{1}{64\pi^2}\text{Str}\left[M^4\left(\ln\frac{M^2}{\Lambda^2} - \frac{3}{2}\right)\right], \tag{82}
$$

at the renormalization point $\Lambda = M_S$ [13][14], where the mass eigenvalues are given by

$$
\begin{array}{ll}
m_{T_\pm}^2 = M_T^2 + (Y_3^U v_u')^2 \pm R_T, & m_{G_\pm}^2 = M_G^2 \pm R_G, \\
M_T^2 = \frac{1}{2}\left(m_{Q_3}^2 + m_{U_3}^2\right) + 5g_x^2(v_s')^2, & M_G^2 = \frac{1}{2}\left(m_G^2 + m_{G^c}^2\right) + (kv_s')^2 - \frac{25}{2}g_x^2(v_s')^2, \\
R_T = \sqrt{(\Delta M_T^2)^2 + (Y_3^U X_T)^2}, & R_G = \sqrt{(\Delta M_G^2)^2 + (kX_G)^2}, \\
\Delta M_T^2 = \frac{1}{2}\left(m_{Q_3}^2 - m_{U_3}^2\right), & \Delta M_G^2 = \frac{1}{2}\left(m_G^2 - m_{G^c}^2\right) + \frac{5}{2}g_x^2(v_s')^2, \\
X_T = \lambda_3 v_s' v_d' - A_3^U v_u', & X_G = \lambda_3 v_u' v_d' - A_k v_s', \\
m_t = Y_3^U v_u', & m_g = kv_s',
\end{array} \tag{83}
$$

where we neglect $O(v_{EW})$ terms in D-term contributions. The mass matrices of stop and G-Higgs are given in following sections (see Eq.(171) and Eq.(236)). By the rotation with

$$
V_H = \frac{1}{v_{EW}}\begin{pmatrix} v_u' & -v_d' \\ v_d' & v_u' \end{pmatrix}, \tag{84}
$$

the $O(1\text{TeV})$ term is eliminated from off-diagonal element of Eq.(78) and we get

$$
\begin{aligned}
m_h^2 &= (V_H^T M^2 V_H)_{11} = (m_h^2)_0 + (m_h^2)_T + (m_h^2)_G, && (85) \\
(m_h^2)_0 &= \left[(\lambda_3 \sin\beta)^2 + \frac{g_Y^2+g_2^2}{2}\cos^2 2\beta + 2g_x^2(2\sin^2\beta + 3\cos^2\beta)^2\right] v_{EW}^2, && (86) \\
(m_h^2)_T &= \frac{3(Y_3^U)^2}{16\pi^2 v_{EW}^2}\left[\frac{(Y_3^U X_T^2)^2}{R_T^2} + 2(Y_3^U(v_u')^2)^2\left(\ln\frac{m_{T_+}^2}{(Y_3^U v_u')^2} + \ln\frac{m_{T_-}^2}{(Y_3^U v_u')^2}\right)\right. \\
&+ \left.\left(\frac{2(Y_3^U v_u')^2 X_T^2}{R_T} - \left[M_T^2 + (Y_3^U v_u')^2\right]\frac{(Y_3^U X_T^2)^2}{2R_T^3}\right)\ln\frac{m_{T_+}^2}{m_{T_-}^2}\right], && (87) \\
(m_h^2)_G &= \frac{9k^2(\lambda_3 v_u' v_d')^2}{8\pi^2 v_{EW}^2}\left[-2\frac{(\Delta M_G^2)^2}{R_G^2} + \frac{M_G^2(\Delta M_G^2)^2}{R_G^3}\ln\frac{m_{G_+}^2}{m_{G_-}^2} + \ln\frac{m_{G_+}^2}{\Lambda^2} + \ln\frac{m_{G_-}^2}{\Lambda^2}\right]. && (88)
\end{aligned}
$$

If we fix the parameters at M_S as given in Table 6, we get

$$
m_h = 125.7, \quad \sqrt{(m_h^2)_0} = 82.7, \quad \sqrt{(m_h^2)_T} = 94.2, \quad \sqrt{(m_h^2)_G} = 9.2 \quad (\text{GeV}). \tag{89}
$$

The 1-loop contribution is dominated by stop and top contributions, this is because we put k small ($k = 0.5$) to intend the mass values of the particles in the loops are within the testable range of LHC at $\sqrt{s} = 14\text{TeV}$ as follows

$$
m_{T_+} = 1882, \quad m_{T_-} = 1178, \quad m_{G_+} = 3908, \quad m_{G_-} = 1737, \quad m_g = 2000 \quad (\text{GeV}). \tag{90}
$$

The value of $\lambda_3 = 0.37$ is tuned to realize observed Higgs mass which is mainly controlled by this parameter through $(\lambda_3 v_{EW}\sin\beta)^2$ and X_T for fixed v_s' and $A_3^U (= A_t)$.

3.3 Chargino and neutralino

At next we consider the higgsinos and the singlinos. The mass matrix of the charged higgsinos is given by

$$\mathcal{L}_C = ((h_1^U)^+,(h_2^U)^+,(h_3^U)^+)\begin{pmatrix}\lambda_1 v_s' & 0 & \lambda_5 v_s c\\ 0 & \lambda_1 v_s' & \lambda_5 v_s s\\ \lambda_4 v_s c & \lambda_4 v_s s & \lambda_3 v_s'\end{pmatrix}\begin{pmatrix}(h_1^D)^-\\ (h_2^D)^-\\ (h_3^D)^-\end{pmatrix} + h.c.. \qquad (91)$$

Since the (3,3) element is much larger than the other $O(\epsilon^2)$ elements, the first and second generation higgsinos decouples and have the same mass $\lambda_1 v_s'$. With the gaugino interaction given as follows

$$\begin{aligned}\mathcal{L}_{\rm gaugino} &= -i\sqrt{2}(H_a^U)^\dagger\left[g_2\sum_{A=1}^3\lambda_2^A T_2^A + \frac{1}{2}g_Y\lambda_Y - 2g_x\lambda_X\right]h_a^U\\ &- i\sqrt{2}(H_a^D)^\dagger\left[g_2\sum_{A=1}^3\lambda_2^A T_2^A - \frac{1}{2}g_Y\lambda_Y - 3g_x\lambda_X\right]h_a^D - i\sqrt{2}(S_a)^\dagger[5g_x\lambda_X]s_a\\ &- \frac{1}{2}M_2\lambda_2^A\lambda_2^A - \frac{1}{2}M_Y\lambda_Y\lambda_Y - \frac{1}{2}M_X\lambda_X\lambda_X + h.c.,\end{aligned} \qquad (92)$$

the third generation charged higgsino mixes with wino and the mass matrix is given by

$$\mathcal{L} \supset ((h_3^U)^+, w^+)\begin{pmatrix}\lambda_3 v_s' & g_2 v_u'\\ g_2 v_d' & M_2\end{pmatrix}\begin{pmatrix}(h_3^D)^-\\ w^-\end{pmatrix} + h.c., \qquad (93)$$

$$w^\pm = \frac{-i}{\sqrt{2}}(\lambda_2^1 \mp i\lambda_2^2). \qquad (94)$$

The mass eigenvalues of charginos are given by

$$\begin{aligned}M\begin{pmatrix}\chi_3^\pm\\ \chi_w^\pm\end{pmatrix} &= \frac{1}{2}\left[(\lambda_3 v_s')^2 + g_2^2(v_u')^2 + g_2^2(v_d')^2 + M_2^2\right]\\ &\pm \sqrt{\frac{1}{4}[(\lambda_3 v_s')^2 + (g_2 v_u')^2 - (g_2 v_d')^2 - M_2^2]^2 + (\lambda_3 g_2 v_s' v_d' + M_2 g_2 v_u')^2},\end{aligned} \qquad (95)$$

$$M(\chi_i^\pm) = \lambda_1 v_s', \qquad (96)$$

where $\chi_3^\pm$ is almost third generation higgsino and $\chi_w^\pm$ is almost wino.

The mass matrix of the neutralinos is divided into two 3×3 matrices and one 6×6 matrix as follows

$$\mathcal{L} \supset -\frac{1}{2}\sum_{i=1,2}((h_i^U)^0,(h_i^D)^0,s_i)\begin{pmatrix}0 & \lambda_1 v_s' & \lambda_5 v_d'\\ \lambda_1 v_s' & 0 & \lambda_4 v_u'\\ \lambda_5 v_d' & \lambda_4 v_u' & 0\end{pmatrix}\begin{pmatrix}(h_i^U)^0\\ (h_i^D)^0\\ s_i\end{pmatrix} - \frac{1}{2}\chi_0^T M_\chi \chi_0, \qquad (97)$$

$$M_\chi = \begin{pmatrix}0 & \lambda_3 v_s' & \lambda_3 v_d' & g_Y v_u'/\sqrt{2} & -g_2 v_u'/\sqrt{2} & -2g_x v_u'\sqrt{2}\\ \lambda_3 v_s' & 0 & \lambda_3 v_u' & -g_Y v_d'/\sqrt{2} & g_2 v_d'/\sqrt{2} & -3g_x v_d'\sqrt{2}\\ \lambda_3 v_d' & \lambda_3 v_u' & 0 & 0 & 0 & 5g_x v_s'\sqrt{2}\\ g_Y v_u'/\sqrt{2} & -g_Y v_d'/\sqrt{2} & 0 & -M_Y & 0 & 0\\ -g_2 v_u'/\sqrt{2} & g_2 v_d'/\sqrt{2} & 0 & 0 & -M_2 & 0\\ -2g_x v_u'\sqrt{2} & -3g_x v_d'\sqrt{2} & 5g_x v_s'\sqrt{2} & 0 & 0 & -M_X\end{pmatrix}, \qquad (98)$$

$$\chi_0^T = ((h_3^U)^0,(h_3^D)^0, s_3, i\lambda_Y, i\lambda_2^3, i\lambda_X). \qquad (99)$$

The mass eigenvalues of these mass matrices are given in Table 7. The common mass of two LSPs is given by the smallest eigenvalue of 3×3 matrix given in Eq.(97). Note that the LEP bound for chargino

$$\lambda_1 v_s' > 100\text{GeV} \quad [15], \qquad (100)$$

must be satisfied. Requiring the coupling constants $\lambda_{4,5}$ do not blow up in $\mu < M_P$, we put upper bound for them as $\lambda_4 < 0.5, \lambda_5 < 0.7$, then the rough estimation of LSP mass is given by

$$M(\chi_{i,1}^0) \sim \frac{2(\lambda_4 v_u')(\lambda_5 v_d')}{\lambda_1 v_s'} < \frac{9000\text{GeV}^2}{\lambda_1 v_s'}, \qquad (101)$$

where $\chi^0_{i,1}$ is almost singlino. To realize density parameter of dark matter $\Omega_{CDM}h^2 = 0.11$, we must tune $M(\chi^0_{i,1}) \sim 30-35$GeV to enhance annihilation cross section. This condition gives upper bound as

$$\lambda_1 v'_s < 300\text{GeV}. \tag{102}$$

This constraint is not consistent with the lower bound as follows

$$m_{\chi_1^\pm} > 295\text{GeV[16]}, \quad m_{\chi_1^\pm} > 330\text{GeV[17]}. \tag{103}$$

Therefore we assume the lightest chargino mass is in the region

$$100 < \lambda_1 v'_s < 140 \quad (\text{GeV}), \tag{104}$$

in which 3-lepton emission is suppressed due to the small mass difference between chargino and neutralino compared with m_Z.

Note that bino-like neutralino can decay into Higgs boson and LSP through the $O(\epsilon^2)$ mixing of Higgs bosons by the interaction

$$\mathcal{L} \supset -i\sqrt{2}(H_1^U)^0\left(\frac{1}{2}g_Y\lambda_Y\right)(h_1^U)^0 \sim O(\epsilon^2)(H_3^U)^0\lambda_Y(h_1^U)^0. \tag{105}$$

4 Quark and Lepton Sector

In this section, we consider the quark and lepton sector and test our model by observed values given as follows, running masses of quarks and charged leptons at $\mu = M_S = 1$TeV [18]

$$\begin{array}{lll} m_u = 1.10^{+0.43}_{-0.37}(\text{MeV}), & m_c = 532\pm 74(\text{MeV}), & m_t = 150.7\pm 3.4(\text{GeV}), \\ m_d = 2.50^{+1.08}_{-1.03}(\text{MeV}), & m_s = 47^{+14}_{-13}(\text{MeV}), & m_b = 2.43\pm 0.08(\text{GeV}), \\ m_e = 0.4959(\text{MeV}), & m_\mu = 104.7(\text{MeV}), & m_\tau = 1780(\text{MeV}), \end{array} \tag{106}$$

CKM matrix elements [19]

$$\begin{array}{lll} |V_{ud}| = 0.97427, & |V_{us}| = 0.22534, & |V_{ub}| = 0.00351, \\ |V_{cd}| = 0.22520, & |V_{cs}| = 0.97344, & |V_{cb}| = 0.0412, \\ |V_{td}| = 0.00867, & |V_{ts}| = 0.0404, & |V_{tb}| = 0.999146, \end{array} \tag{107}$$

and neutrino masses and MNS mixing angles [19]

$$\Delta m^2_{21} = m^2_{\nu_2} - m^2_{\nu_1} = (7.58^{+0.22}_{-0.26}) \times 10^{-5} \quad (\text{eV}^2), \tag{108}$$

$$\Delta m^2_{32} = \left|m^2_{\nu_3} - m^2_{\nu_2}\right| = (2.35^{+0.12}_{-0.09}) \times 10^{-3} \quad (\text{eV}^2), \tag{109}$$

$$V_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\sin^2\theta_{12} = 0.306^{+0.018}_{-0.015}, \quad \sin^2\theta_{23} = 0.42^{+0.08}_{-0.03}, \quad \sin^2\theta_{13} = 0.021^{+0.007}_{-0.008}. \tag{110}$$

After that we estimate the flavor changing process induced by sfermion exchange.

4.1 Quark sector

The superpotential of up-type quark sector is given by

$$\begin{aligned} W_U &= Y_3^U H_3^U Q_3 U_3^c + \epsilon^2 Y_2^U H_3^U[Q_1 s_2 + Q_2 c_2]U_3^c + \epsilon^3 Y_4^U H_3^U[Q_1 c + Q_2 s]U_2^c + \epsilon^4 Y_1^U H_3^U Q_3 U_1^c \\ &+ \epsilon^6\left\{Y_5^U H_3^U[Q_1 s_2 + Q_2 c_2]U_1^c + Y_6^U H_3^U[s_3(Q_1 c + Q_2 s)]U_1^c - Y_7^U H_3^U[c_3(Q_1 s - Q_2 c)]U_1^c\right\}, \end{aligned} \tag{111}$$

from which we get up-type quark mass matrix as

$$M_u = \begin{pmatrix} \epsilon^6(Y_5^U s_2 + Y_6^U s_3 c - Y_7^U c_3 s) & \epsilon^3 Y_4^U c & \epsilon^2 Y_2^U s_2 \\ \epsilon^6(Y_5^U c_2 + Y_6^U s_3 s + Y_7^U c_3 c) & \epsilon^3 Y_4^U s & \epsilon^2 Y_2^U c_2 \\ \epsilon^4 Y_1^U & 0 & Y_3^U \end{pmatrix} v'_u = \begin{pmatrix} \epsilon^6 & \epsilon^3 & \epsilon^2 \\ \epsilon^6 & \epsilon^3 & \epsilon^2 \\ \epsilon^4 & 0 & 1 \end{pmatrix} v'_u. \tag{112}$$

Note that there is dangerous VEV direction such as $\theta = \frac{\pi}{6}$. In this direction the matrix given in Eq.(112) is given by

$$M_u = \begin{pmatrix} \epsilon^6(Y_5^U + Y_6^U)c & \epsilon^3 Y_4^U c & \epsilon^2 Y_2^U c \\ \epsilon^6(Y_5^U + Y_6^U)s & \epsilon^3 Y_4^U s & \epsilon^2 Y_2^U s \\ \epsilon^4 Y_1^U & 0 & Y_3^U \end{pmatrix} v'_u, \tag{113}$$

which has zero eigenvalue. In the same way, the down type quark masses are given by

$$\begin{aligned} W_D &= \epsilon^4 \left\{ Y_2^D H_3^D [s_3(Q_1 c + Q_2 s)] D_3^c + Y_4^D H_3^D (Q_1 s_2 + Q_2 c_2) D_3^c + Y_9^D H_3^D [c_3(-Q_1 s + Q_2 c)] D_3^c \right. \\ &+ Y_5^D H_3^D [s_3(-Q_1 s + Q_2 c)] D_2^c + Y_6^D H_3^D (-Q_1 c_2 + Q_2 s_2) D_2^c + \left. Y_{10}^D H_3^D [c_3(Q_1 c + Q_2 s)] D_2^c \right\} \\ &+ \epsilon^5 \left\{ Y_8^D H_3^D [Q_1 c + Q_2 s] D_1^c + Y_7^D H_3^D [s_3(Q_1 s_2 + Q_2 c_2)] D_1^c \right. \\ &+ \left. Y_{11}^D H_3^D [c_3(-Q_1 c_2 + Q_2 s_2)] D_1^c \right\} + \epsilon^2 Y_3^D H_3^D Q_3 D_3^c + \epsilon^3 Y_1^D s_3 H_3^D Q_3 D_1^c, \end{aligned} \tag{114}$$

from which we get down-type quark mass matrix as follows

$$\begin{aligned} & M_d / v'_d \\ = & \begin{pmatrix} \epsilon^5(Y_7^D s_3 s_2 + Y_8^D c - Y_{11}^D c_3 c_2) & \epsilon^4(-Y_5^D s_3 s - Y_6^D c_2 + Y_{10}^D c_3 c) & \epsilon^4(Y_2^D s_3 c + Y_4^D s_2 - Y_9^D c_3 s) \\ \epsilon^5(Y_7^D s_3 c_2 + Y_8^D s + Y_{11}^D c_3 s_2) & \epsilon^4(Y_5^D s_3 c + Y_6^D s_2 + Y_{10}^D c_3 s) & \epsilon^4(Y_2^D s_3 s + Y_4^D c_2 + Y_9^D c_3 c) \\ \epsilon^4 Y_1^D & 0 & \epsilon^2 Y_3^D \end{pmatrix} \\ = & \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & 0 & \epsilon^2 \end{pmatrix}. \end{aligned} \tag{115}$$

The effects of flavor violation appear not only in superpotential but also in Kähler potential as follows

$$\begin{aligned} K(U^c) &= |U_1^c|^2 + |U_2^c|^2 + |U_3^c|^2 + \left\{ \frac{(E_3 U_1^c)^* U_2^c}{M_P^3} + \frac{(E_2^2 U_1^c)^* U_3^c}{M_P^4} + \frac{(E_3 U_2^c)^* (E_2 U_3^c)}{M_P^5} + h.c. \right\} \\ &= ((U_1^c)^*, (U_2^c)^*, (U_3^c)^*) \begin{pmatrix} 1 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & 1 & \epsilon^5 \\ \epsilon^4 & \epsilon^5 & 1 \end{pmatrix} \begin{pmatrix} U_1^c \\ U_2^c \\ U_3^c \end{pmatrix}, \end{aligned} \tag{116}$$

$$\begin{aligned} K(D^c) &= |D_1^c|^2 + |D_2^c|^2 + |D_3^c|^2 + \left\{ \frac{(V_2 D_1^c)^* \cdot (V_1 D_2^c)}{M_P^3} + \frac{(V_2 D_1^c)^* \cdot (V_1 D_3^c)}{M_P^3} + \frac{(E_3 D_2^c)^* (P_3 D_3^c)}{M_P^6} + h.c. \right\} \\ &= ((D_1^c)^*, (D_2^c)^*, (D_3^c)^*) \begin{pmatrix} 1 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & 1 & \epsilon^6 \\ \epsilon^3 & \epsilon^6 & 1 \end{pmatrix} \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \end{pmatrix}, \end{aligned} \tag{117}$$

$$\begin{aligned} K(Q) &= (|Q_1|^2 + |Q_2|^2) + |Q_3|^2 + \left\{ \frac{(V_2 \cdot Q)^* Q_3}{M_P^2} + \frac{|V_1 \cdot Q|^2}{M_P^2} + \cdots + h.c. \right\} \\ &= (Q_1^*, Q_2^*, Q_3^*) \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}, \end{aligned} \tag{118}$$

where dot in $X \cdot Y$ means inner product of two S_4-doublets X, Y. Therefore, a superfield redefinition has to be performed in order to get canonical kinetic terms as follows [20]

$$\begin{pmatrix} U_1^c \\ U_2^c \\ U_3^c \end{pmatrix} = V_K(U) \begin{pmatrix} (U_1^c)' \\ (U_2^c)' \\ (U_3^c)' \end{pmatrix}, \quad V_K(U) = \begin{pmatrix} 1 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & 1 & \epsilon^5 \\ \epsilon^4 & \epsilon^5 & 1 \end{pmatrix}, \tag{119}$$

$$\begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \end{pmatrix} = V_K(D) \begin{pmatrix} (D_1^c)' \\ (D_2^c)' \\ (D_3^c)' \end{pmatrix}, \quad V_K(D) = \begin{pmatrix} 1 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & 1 & \epsilon^6 \\ \epsilon^3 & \epsilon^6 & 1 \end{pmatrix}, \tag{120}$$

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} = V_K(Q) \begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{pmatrix}, \quad V_K(Q) = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}, \tag{121}$$

by which the mass matrices given above are transformed into

$$M'_u = V_K^T(Q)\begin{pmatrix}\epsilon^6 & \epsilon^3 & \epsilon^2\\ \epsilon^6 & \epsilon^3 & \epsilon^2\\ \epsilon^4 & 0 & 1\end{pmatrix} v'_u V_K(U) = \begin{pmatrix}\epsilon^6 & \epsilon^3 & \epsilon^2\\ \epsilon^6 & \epsilon^3 & \epsilon^2\\ \epsilon^4 & \epsilon^5 & 1\end{pmatrix} v'_u, \tag{122}$$

$$M'_d = V_K^T(Q)\begin{pmatrix}\epsilon^5 & \epsilon^4 & \epsilon^4\\ \epsilon^5 & \epsilon^4 & \epsilon^4\\ \epsilon^3 & 0 & \epsilon^2\end{pmatrix} v'_d V_K(D) = \begin{pmatrix}\epsilon^5 & \epsilon^4 & \epsilon^4\\ \epsilon^5 & \epsilon^4 & \epsilon^4\\ \epsilon^3 & \epsilon^6 & \epsilon^2\end{pmatrix} v'_d. \tag{123}$$

These matrices are diagonalized as follows

$$M'_u = L_u M_u^{\rm diag} R_u^\dagger = \begin{pmatrix}1 & 1 & \epsilon^2\\ 1 & 1 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & 1\end{pmatrix}\begin{pmatrix}\epsilon^6 & 0 & 0\\ 0 & \epsilon^3 & 0\\ 0 & 0 & 1\end{pmatrix} v'_u \begin{pmatrix}1 & \epsilon^3 & \epsilon^4\\ \epsilon^3 & 1 & \epsilon^5\\ \epsilon^4 & \epsilon^5 & 1\end{pmatrix}, \tag{124}$$

$$M'_d = L_d M_d^{\rm diag} R_d^\dagger = \begin{pmatrix}1 & 1 & \epsilon^2\\ 1 & 1 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & 1\end{pmatrix}\begin{pmatrix}\epsilon^5 & 0 & 0\\ 0 & \epsilon^4 & 0\\ 0 & 0 & \epsilon^2\end{pmatrix} v'_d \begin{pmatrix}1 & \epsilon^3 & \epsilon\\ \epsilon^3 & 1 & \epsilon^4\\ \epsilon & \epsilon^4 & 1\end{pmatrix}. \tag{125}$$

Therefore Yukawa hierarchies are given by

$$\begin{aligned}&Y_u(M_P) = \epsilon^6, \quad Y_c(M_P) = \epsilon^3, \quad Y_t(M_P) = Y_3^U(M_P) = 1,\\ &Y_d(M_P) = \epsilon^5, \quad Y_s(M_P) = \epsilon^4, \quad Y_b(M_P) = \epsilon^2.\end{aligned} \tag{126}$$

On the other hand, observed values Eq.(106) give

$$Y_u(M_P) = \frac{1}{5.1}\left(\frac{1.10\times 10^{-3}}{150.7}\right) = 1.4\epsilon^6, \tag{127}$$

$$Y_c(M_P) = \frac{1}{5.1}\left(\frac{532\times 10^{-3}}{150.7}\right) = 0.69\epsilon^3, \tag{128}$$

$$Y_t(M_P) = 0.28, \tag{129}$$

$$Y_d(M_P) = \frac{1}{7.2}\left(\frac{2.50\times 10^{-3}}{87}\right) = 0.40\epsilon^5, \tag{130}$$

$$Y_s(M_P) = \frac{1}{7.2}\left(\frac{47\times 10^{-3}}{87}\right) = 0.75\epsilon^4, \tag{131}$$

$$Y_d(M_P) = \frac{1}{7.2}\left(\frac{2.43}{87}\right) = 0.39\epsilon^2, \tag{132}$$

which give good agreement with Eq.(126). Where the renormalization factors

$$\sqrt{\frac{\alpha_u(M_S)}{\alpha_u(M_P)}} = 5.1, \quad \sqrt{\frac{\alpha_d(M_S)}{\alpha_d(M_P)}} = 7.2, \tag{133}$$

and $Y_t(M_P) = 0.28$ are calculated based on RGEs given in appendix A. CKM matrix is given by

$$V_{CKM} = L_u^\dagger L_d = \begin{pmatrix}1 & 1 & \epsilon^2\\ 1 & 1 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & 1\end{pmatrix}, \tag{134}$$

which is consistent with Eq.(107).

Note that the $Z_2^{(2)}$ breaking induces generation mixing in Higgs bosons then Yukawa interactions are modified as follows

$$-\mathcal{L} = Y_{ij}^U(H_3^U + \epsilon^2 H_1^U + \epsilon^2 H_2^U)q_i u_j^c + Y_{ij}^D(H_3^D + \epsilon^2 H_1^D + \epsilon^2 H_2^D)q_i d_j^c. \tag{135}$$

Since these Yukawa coupling matrices are diagonalized in the basis that the quark mass matrices are diagonalized, the extra Higgs boson exchange do not contribute to the flavor changing processes.

4.2 Lepton sector

With the straightforward calculation, the mass matrices of lepton sector are given as follows. From the superpotentials

$$
\begin{aligned}
W_E &= H_3^D(L_1,L_2,L_3)\begin{pmatrix} \epsilon^5(Y_7^E c + Y_8^E s_3 s_2 - Y_{10}^E c_3 c_2) & \epsilon^3 Y_5^E c & \epsilon^2 Y_4^E s_2 \\ \epsilon^5(Y_7^E s + Y_8^E s_3 c_2 + Y_{10}^E c_3 s_2) & \epsilon^3 Y_5^E s & \epsilon^2 Y_4^E c_2 \\ \epsilon^5 Y_1^E s_3 & \epsilon^3 Y_2^E s_3 & \epsilon^2 Y_3^E \end{pmatrix}\begin{pmatrix} E_1^c \\ E_2^c \\ E_3^c \end{pmatrix}, \qquad (136) \\
W_N &= \epsilon^3 H_1^U(L_1,L_2,L_3)\begin{pmatrix} 0 & Y_1^N s_3 + Y_4^N c s_2 + \cdots & Y_5^N s_3 + Y_8^N c s_2 + \cdots \\ 0 & -Y_2^N c_3 + Y_4^N c c_2 + \cdots & -Y_6^N c_3 + Y_8^N c c_2 + \cdots \\ 0 & Y_3^N c & Y_7^N c \end{pmatrix}\begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix} \\
&+ \epsilon^3 H_2^U(L_1,L_2,L_3)\begin{pmatrix} 0 & Y_2^N c_3 + Y_4^N s s_2 + \cdots & Y_6^N c_3 + Y_8^N s s_2 + \cdots \\ 0 & Y_1^N s_3 + Y_4^N s c_2 + \cdots & Y_5^N s_3 + Y_8^N s c_2 + \cdots \\ 0 & Y_3^N s & Y_7^N s \end{pmatrix}\begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix}, \qquad (137) \\
W_R &= \frac{1}{M_P}(\Phi_1^2+\Phi_2^2+\Phi_3^2)\left[Y_{11}^N N_1^c N_1^c + Y_{22}^N N_2^c N_2^c + Y_{33}^N N_3^c N_3^c + Y_{23}^N N_2^c N_3^c\right], \qquad (138)
\end{aligned}
$$

we get original mass matrices as follows

$$
M_e = \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix} v_d', \quad M_D = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \epsilon^2 v_u, \quad M_M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{V^2}{M_P}. \qquad (139)
$$

Redefining the Kähler potential given by

$$
\begin{aligned}
K(E^c) &= |E_1^c|^2 + |E_2^c|^2 + |E_3^c|^2 + \left\{\frac{(E_2 E_1^c)^* E_2^c}{M_P^2} + \frac{(E_3 E_1^c)^* E_3^c}{M_P^3} + \frac{(V_2 E_2^c)^* \cdot (V_1 E_3^c)}{M_P^3} + h.c.\right\} \\
&= ((E_1^c)^*, (E_2^c)^*, (E_3^c)^*)\begin{pmatrix} 1 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & 1 & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}\begin{pmatrix} E_1^c \\ E_2^c \\ E_3^c \end{pmatrix}, \qquad (140) \\
K(L) &= (|L_1|^2 + |L_2|^2) + |L_3|^2 \\
&+ \left\{\frac{(L_1 D_2 + L_2 D_1)^*(D_1 L_3) + (L_1 D_1 - L_2 D_2)^*(D_2 L_3)}{M_P^2} + \frac{|L \cdot V_1|^2}{M_P^2} + \cdots + h.c.\right\} \\
&= (L_1^*, L_2^*, L_3^*)\begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix}, \qquad (141) \\
K(N^c) &= |N_1^c|^2 + |N_2^c|^2 + |N_3^c|^2 + \left\{\frac{(V_1 N_2^c)^* \cdot (V_1 N_3^c)}{M_P^2} + h.c.\right\} \\
&= ((N_1^c)^*, (N_2^c)^*, (N_3^c)^*)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon^2 \\ 0 & \epsilon^2 & 1 \end{pmatrix}\begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix}, \qquad (142)
\end{aligned}
$$

by the superfields redefinition as

$$
\begin{aligned}
\begin{pmatrix} E_1^c \\ E_2^c \\ E_3^c \end{pmatrix} &= V_K(E)\begin{pmatrix} (E_1^c)' \\ (E_2^c)' \\ (E_3^c)' \end{pmatrix}, \quad V_K(E) = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & 1 & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}, \qquad (143) \\
\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} &= V_K(L)\begin{pmatrix} L_1' \\ L_2' \\ L_3' \end{pmatrix}, \quad V_K(L) = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}, \qquad (144) \\
\begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix} &= V_K(N)\begin{pmatrix} (N_1^c)' \\ (N_2^c)' \\ (N_3^c)' \end{pmatrix}, \quad V_K(N) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon^2 \\ 0 & \epsilon^2 & 1 \end{pmatrix}, \qquad (145)
\end{aligned}
$$

the modified mass matrices are given by

$$
M_e' = V_K^T(L)\begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix} v_d' V_K(E) = \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix} v_d', \qquad (146)
$$

$$M'_D = V_K^T(L)\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\epsilon^3 v_u V_K(N) = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\epsilon^3 v_u, \tag{147}$$

$$M'_M = V_K^T(N)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\frac{V^2}{M_P}V_K(N) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\frac{V^2}{M_P}. \tag{148}$$

The mixing matrices of charged leptons are given by

$$M'_e = L_e M_e^{\mathrm{diag}} R_e^\dagger = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \epsilon^5 & 0 & 0 \\ 0 & \epsilon^3 & 0 \\ 0 & 0 & \epsilon^2 \end{pmatrix} v'_d \begin{pmatrix} 1 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & 1 & \epsilon \\ \epsilon^3 & \epsilon & 1 \end{pmatrix}. \tag{149}$$

The Yukawa hierarchy of charged leptons gives good agreement with the experimental values given by

$$Y_e(M_P) = \frac{1}{1.9}\left(\frac{0.496\times 10^{-3}}{87}\right) = 0.30\epsilon^5, \tag{150}$$

$$Y_\mu(M_P) = \frac{1}{1.9}\left(\frac{105\times 10^{-3}}{87}\right) = 0.64\epsilon^3, \tag{151}$$

$$Y_\tau(M_P) = \frac{1}{1.9}\left(\frac{1.78}{87}\right) = 1.08\epsilon^2, \tag{152}$$

where the used value of renormalization factor

$$\sqrt{\frac{\alpha_e(M_S)}{\alpha_e(M_P)}} = 1.9 \tag{153}$$

is calculated based on RGEs given in appendix A.

The neutrino seesaw mass matrix is given by

$$M_\nu = (M'_D)(M'_M)^{-1}(M'_D)^T = m_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad m_\nu = \frac{(\epsilon^3 v_u)^2}{M_R} = O(0.01\mathrm{eV}), \quad M_R = \frac{V^2}{M_P}, \tag{154}$$

which has one zero eigenvalue because one RHN n_1^c does not couple to left-handed leptons. Therefore mixing matrix and mass eigenvalues are given as follows

$$L_\nu^T M_\nu L_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \tag{155}$$

$$L_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{156}$$

$$m_{\nu_1} = 0, \qquad m_{\nu_2} = \sqrt{m_{21}^2} = 0.87\times 10^{-2}, \quad m_{\nu_3} \simeq \sqrt{m_{32}^2} = 4.8\times 10^{-2} \quad (\mathrm{eV}), \tag{157}$$

and MNS matrix is given by

$$V_{MNS} = L_e^\dagger L_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{158}$$

With the recent experimental value of $|\sin\theta_{13}| \sim 0.14$ [21], MNS matrix is given by

$$V_{MNS} = \begin{pmatrix} 0.64 & 0.55 & 0.14 \\ 0.42 & 0.64 & 0.65 \\ 0.36 & 0.55 & 0.76 \end{pmatrix}, \tag{159}$$

which is consistent with Eq.(158).

4.3 Squark and slepton sector

Sfermion mass matrices are given as follows

$$
\begin{aligned}
-\mathcal{L} &\supset \sum_{X=Q,U^c,D^c,L,E^c} X_a^*[M^2(X)]_{ab}X_b \\
&- [H_3^U Q_a A(U)_{ab} U_b^c + H_3^D Q_a A(D)_{ab} D_b^c + H_3^D L_a A(E)_{ab} E_b^c + h.c.] + V_F + V_D, && (160)\\
M^2(Q) &= \begin{pmatrix} m_Q^2 & m^2\epsilon^2 & m^2\epsilon^2 \\ m^2\epsilon^2 & m_Q^2 & m^2\epsilon^2 \\ m^2\epsilon^2 & m^2\epsilon^2 & m_{Q_3}^2 \end{pmatrix}, && (161)\\
M^2(U^c) &= \begin{pmatrix} m_{U_1^c}^2 & m^2\epsilon^3 & m^2\epsilon^4 \\ m^2\epsilon^3 & m_{U_2^c}^2 & m^2\epsilon^5 \\ m^2\epsilon^4 & m^2\epsilon^5 & m_{U_3^c}^2 \end{pmatrix}, && (162)\\
M^2(D^c) &= \begin{pmatrix} m_{D_1^c}^2 & m^2\epsilon^3 & m^2\epsilon^3 \\ m^2\epsilon^3 & m_{D_2^c}^2 & m^2\epsilon^6 \\ m^2\epsilon^3 & m^2\epsilon^6 & m_{D_3^c}^2 \end{pmatrix}, && (163)\\
M^2(L) &= \begin{pmatrix} m_L^2 & m^2\epsilon^2 & m^2\epsilon^2 \\ m^2\epsilon^2 & m_L^2 & m^2\epsilon^2 \\ m^2\epsilon^2 & m^2\epsilon^2 & m_{L_3}^2 \end{pmatrix}, && (164)\\
M^2(E^c) &= \begin{pmatrix} m_{E_1^c}^2 & m^2\epsilon^2 & m^2\epsilon^3 \\ m^2\epsilon^2 & m_{E_2^c}^2 & m^2\epsilon^3 \\ m^2\epsilon^3 & m^2\epsilon^3 & m_{E_3^c}^2 \end{pmatrix}, && (165)\\
A(U) &= \begin{pmatrix} \epsilon^6 Y^U A^U & \epsilon^3 Y^U A^U & \epsilon^2 Y^U A^U \\ \epsilon^6 Y^U A^U & \epsilon^3 Y^U A^U & \epsilon^2 Y^U A^U \\ \epsilon^4 Y^U A^U & 0 & Y_3^U A_3^U \end{pmatrix}, && (166)\\
A(D) &= \begin{pmatrix} \epsilon^5 Y^D A^D & \epsilon^4 Y^D A^D & \epsilon^4 Y^D A^D \\ \epsilon^5 Y^D A^D & \epsilon^4 Y^D A^D & \epsilon^4 Y^D A^D \\ \epsilon^3 Y^D A^D & 0 & \epsilon^2 Y^D A^D \end{pmatrix}, && (167)\\
A(E) &= \begin{pmatrix} \epsilon^5 Y^E A^E & \epsilon^3 Y^E A^E & \epsilon^2 Y^E A^E \\ \epsilon^5 Y^E A^E & \epsilon^3 Y^E A^E & \epsilon^2 Y^E A^E \\ \epsilon^5 Y^E A^E & \epsilon^3 Y^E A^E & \epsilon^2 Y^E A^E \end{pmatrix}, && (168)\\
V_F &= |Y_3^U H_3^U Q_3|^2 + |Y_3^U H_3^U U_3^c|^2 + |Y_3^U Q_3 U_3^c + \lambda_3 S_3 H_3^D|^2, && (169)\\
V_D &= \frac{1}{2} g_x^2 \left[5|S_3|^2 + \sum_{a=1}^{3} (|Q_a|^2 + |U_a^c|^2 + 2|D_a^c|^2 + 2|L_a|^2 + |E_a^c|^2) \right]^2, && (170)
\end{aligned}
$$

where $m = O(10^3\text{GeV})$ and the contributions from F-terms are neglected except for top-Yukawa contributions and the contributions from D-terms are neglected except for the contributions from S_3.

After the redefinition of Kähler potential and the diagonalization of Yukawa matrices, sfermion masses are given as follows

$$
\begin{aligned}
-\mathcal{L} &\supset \sum_{i=1}^{2} \left(m_{U_i^c}^2 + 5g_x^2 (v_s')^2 \right) |U_i^c|^2 + \sum_{a=1}^{3} \left(m_{D_a^c}^2 + 10 g_x^2 (v_s')^2 \right) |D_a^c|^2 + \sum_{i=1}^{2} \left(m_Q^2 + 5g_x^2 (v_s')^2 \right) |Q_i|^2 \\
&+ \left(m_{Q_3}^2 + 5g_x^2 (v_s')^2 \right) |D_3|^2 + \sum_{i=1}^{2} \left(m_L^2 + 10 g_x^2 (v_s')^2 \right) |L_i|^2 \\
&+ \left(m_{L_3}^2 + 10 g_x^2 (v_s')^2 \right) |L_3|^2 + \sum_{a=1}^{3} \left(m_{E_a^c}^2 + 5 g_x^2 (v_s')^2 \right) |E_a^c|^2 \\
&+ (U_3^*, U_3^c) \begin{pmatrix} m_{Q_3}^2 + (Y_3^U v_u')^2 + 5g_x^2 (v_s')^2 & Y_3^U \lambda_3 v_s' v_d' - A_3^U Y_3^U v_u' \\ Y_3^U \lambda_3 v_s' v_d' - A_3^U Y_3^U v_u' & m_{U_3}^2 + (Y_3^U v_u')^2 + 5 g_x^2 (v_s')^2 \end{pmatrix} \begin{pmatrix} U_3 \\ (U_3^c)^* \end{pmatrix} \\
&+ m^2 U_a^* (\delta_{LL}^U)_{ab} U_b + m^2 D_a^* (\delta_{LL}^D)_{ab} D_b + m^2 (U^c)_a^* (\delta_{RR}^U)_{ab} U_b^c + m^2 (D^c)_a^* (\delta_{RR}^D)_{ab} D_b^c
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad m^2 E_a^*(\delta_{LL}^E)_{ab} E_b + m^2 N_a^*(\delta_{LL}^N)_{ab} N_b + m^2 (E^c)_a^*(\delta_{RR}^E)_{ab} E_b^c \\
&- \quad m^2 \left\{ U_a(\delta_{LR}^U)_{ab} U_b^c + D_a(\delta_{LR}^D)_{ab} D_b^c + E_a(\delta_{LR}^E)_{ab} E_b^c + h.c. \right\},
\end{aligned}
\tag{171}
$$

$$
\delta_{LL}^U = \frac{1}{m^2}[L_u^\dagger V_K^\dagger(Q) M^2(Q) V_K(Q) L_u]_{\text{off diagonal}} = \begin{pmatrix} 0 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 0 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 0 \end{pmatrix},
\tag{172}
$$

$$
\delta_{LL}^D = \frac{1}{m^2}[L_d^\dagger V_K^\dagger(Q) M^2(Q) V_K(Q) L_d]_{\text{off diagonal}} = \begin{pmatrix} 0 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 0 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 0 \end{pmatrix},
\tag{173}
$$

$$
\delta_{LL}^E = \frac{1}{m^2}[L_e^\dagger V_K^\dagger(L) M^2(L) V_K(L) L_e]_{\text{off diagonal}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},
\tag{174}
$$

$$
\delta_{RR}^U = \frac{1}{m^2}[R_u^\dagger V_K^\dagger(U) M^2(U) V_K(U) R_u]_{\text{off diagonal}} = \begin{pmatrix} 0 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 0 & \epsilon^5 \\ \epsilon^4 & \epsilon^5 & 0 \end{pmatrix},
\tag{175}
$$

$$
\delta_{RR}^D = \frac{1}{m^2}[R_d^\dagger V_K^\dagger(D) M^2(D) V_K(D) R_d]_{\text{off diagonal}} = \begin{pmatrix} 0 & \epsilon^3 & \epsilon \\ \epsilon^3 & 0 & \epsilon^4 \\ \epsilon & \epsilon^4 & 0 \end{pmatrix},
\tag{176}
$$

$$
\delta_{LR}^U = \frac{1}{m^2}[L_u^T V_K^T(Q) A(U) V_K(U) R_u]_{A_3^U=0} = \frac{v_u' Y^U A^U}{m^2} \begin{pmatrix} \epsilon^6 & \epsilon^3 & \epsilon^2 \\ \epsilon^6 & \epsilon^3 & \epsilon^2 \\ \epsilon^4 & \epsilon^5 & 0 \end{pmatrix},
\tag{177}
$$

$$
\delta_{LR}^D = \frac{1}{m^2}[L_d^T V_K^T(Q) A(D) V_K(D) R_d] = \frac{v_d' Y^D A^D}{m^2} \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^6 & \epsilon^2 \end{pmatrix},
\tag{178}
$$

$$
\delta_{LR}^E = \frac{1}{m^2}[L_e^T V_K^T(L) A(E) V_K(E) R_e] = \frac{v_d' Y^E A^E}{m^2} \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix},
\tag{179}
$$

where the off diagonal parts are extracted except for stop mass matrix and $\delta_{LL}^N, \delta_{RR}^E$ are omitted.

4.4 Flavor and CP violation

The off diagonal elements of sfermion mass matrices contribute to flavor and CP violation through the sfermion exchange, on which are imposed severe constraints. Based on the estimations of the flavor and CP violations with the mass insertion approximation, the upper bounds for each elements are given in Table 4, where $M_Q = M(\text{gluino}) = M(\text{squark}), M_L = M(\text{slepton}) = M(\text{photino})$ are assumed [22]. Note that there is another suppression factor in δ_{LR}^X as

$$
\frac{v_{u,d}'}{m} \sim \epsilon.
\tag{180}
$$

The most stringent bound for M_L is given by $\mu \to e\gamma$ as

$$
1 < 1.5 \times 10^{-2} \left(\frac{M_L}{300\text{GeV}} \right)^2 \quad : \quad M_L > 2250\text{GeV},
\tag{181}
$$

and the one for M_Q is given by ϵ_K as

$$
\epsilon^{2.5} = 3 \times 10^{-3} < 4.4 \times 10^{-4} \left(\frac{M_Q}{1000\text{GeV}} \right) \quad : \quad M_Q > 6820\text{GeV}.
\tag{182}
$$

Note that if $Q_{1,2}$ were S_4-singlets, then $(\delta_{LL}^U)_{12}$ would be $O(1)$ and the most stringent bound for M_Q would be given by

$$
1 < 6.4 \times 10^{-3} \left(\frac{M_Q}{1000\text{GeV}} \right) \quad : \quad M_Q > 156\text{TeV}.
\tag{183}
$$

Comparing Eq.(182) and Eq.(183), one can see that S_4 softens the SUSY flavor problem very efficiently.

Before ending this section, we discuss the problem of a complex flavon VEV. If the relative phase of two VEVs $\langle D_1 \rangle, \langle D_2 \rangle$ exists, we must include

$$K(D^c) \quad \supset \quad \left\{ \frac{[-D_1^* D_2 + D_2^* D_1](D_2^c)^*(D_3^c)}{M_P^2} + h.c. \right\}, \tag{184}$$

in Kähler potential, then redefinition of superfields are modified as follows

$$\begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \end{pmatrix} = K'(D) \begin{pmatrix} (D_1^c)' \\ (D_2^c)' \\ (D_3^c)' \end{pmatrix}, \quad V_K(D) = \begin{pmatrix} 1 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}. \tag{185}$$

Therefore the mass matrix and mixing matrix of down quark sector and off-diagonal matrix of squarks are modified as follows

$$M_d' = \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^4 & \epsilon^2 \end{pmatrix}, \quad R_d' = \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon^2 \\ \epsilon & \epsilon^2 & 1 \end{pmatrix}, \quad \delta_{RR}^D = \begin{pmatrix} 0 & \epsilon & \epsilon \\ \epsilon & 0 & \epsilon^2 \\ \epsilon & \epsilon^2 & 0 \end{pmatrix}. \tag{186}$$

As the result, the most stringent bound for M_Q is changed into $M_Q > 68$TeV. This suggests new mechanism is needed to suppress CP violation. We leave this problem for future work.

observable	parameter	order	upper bound	coefficient
Δm_K	$\sqrt{(\delta_{LL}^D)_{12}(\delta_{RR}^D)_{12}}$ $(\delta_{LL}^D)_{12}$ $(\delta_{LR}^D)_{12}$	$\epsilon^{2.5}$ ϵ^2 ϵ^4	5.6×10^{-3} 8.0×10^{-2} 8.8×10^{-3}	$\times \left(\frac{M_Q}{1000GeV}\right) \sqrt{\frac{(\Delta m_K)_{exp}}{3.49\times 10^{-12} MeV}}$
Δm_B	$\sqrt{(\delta_{LL}^D)_{13}(\delta_{RR}^D)_{13}}$ $(\delta_{LL}^D)_{13}$ $(\delta_{LR}^D)_{13}$	$\epsilon^{1.5}$ ϵ^2 ϵ^3	3.4×10^{-2} 1.9×10^{-1} 6.3×10^{-2}	$\times \left(\frac{M_Q}{1000GeV}\right) \sqrt{\frac{(\Delta m_B)_{exp}}{3.38\times 10^{-10} MeV}}$
Δm_D	$\sqrt{(\delta_{LL}^U)_{12}(\delta_{RR}^U)_{12}}$ $(\delta_{LL}^U)_{12}$ $(\delta_{LR}^U)_{12}$	ϵ^2 ϵ^2 ϵ^3	1.1×10^{-2} 6.2×10^{-2} 1.9×10^{-2}	$\times \left(\frac{M_Q}{1000GeV}\right) \sqrt{\frac{(\Delta m_B)_{exp}}{1.26\times 10^{-11} MeV}}$
ϵ_K	$\sqrt{\lvert \mathrm{Im}[(\delta_{LL}^D)_{12}(\delta_{RR}^D)_{12}] \rvert}$ $\sqrt{\lvert \mathrm{Im}(\delta_{LL}^D)_{12}^2 \rvert}$ $\sqrt{\lvert \mathrm{Im}(\delta_{LR}^D)_{12}^2 \rvert}$	$\epsilon^{2.5}$ ϵ^2 ϵ^4	4.4×10^{-4} 6.4×10^{-3} 7.0×10^{-4}	$\times \left(\frac{M_Q}{1000GeV}\right) \sqrt{\frac{(\epsilon_K)_{exp}}{2.24\times 10^{-3}}}$
$\mu \to e\gamma$	$(\delta_{LL}^E)_{12}$ $(\delta_{LR}^E)_{12}$	1 ϵ^3	1.5×10^{-2} 3.4×10^{-6}	$\times \left(\frac{M_L}{300GeV}\right)^2 \sqrt{\frac{(BR(\mu\to e\gamma))_{exp}}{2.4\times 10^{-12}}}$
$\tau \to e\gamma$	$(\delta_{LL}^E)_{13}$ $(\delta_{LR}^E)_{13}$	1 ϵ^2	4.3 1.6×10^{-2}	$\times \left(\frac{M_L}{300GeV}\right)^2 \sqrt{\frac{(BR(\tau\to e\gamma))_{exp}}{3.3\times 10^{-8}}}$
$\tau \to \mu\gamma$	$(\delta_{LL}^E)_{23}$ $(\delta_{LR}^E)_{23}$	1 ϵ^2	4.9 1.8×10^{-2}	$\times \left(\frac{M_L}{300GeV}\right)^2 \sqrt{\frac{(BR(\tau\to \mu\gamma))_{exp}}{4.4\times 10^{-8}}}$
d_n	$\lvert \mathrm{Im}(\delta_{LR}^U)_{11} \rvert$ $\lvert \mathrm{Im}(\delta_{LR}^D)_{11} \rvert$	ϵ^6 ϵ^5	3.1×10^{-6} 1.6×10^{-6}	$\times \left(\frac{M_Q}{1000GeV}\right) \left(\frac{(d_n)_{exp}}{2.9\times 10^{-26} ecm}\right)$
d_e	$\lvert \mathrm{Im}(\delta_{LR}^E)_{11} \rvert$	ϵ^5	1.7×10^{-7}	$\times \left(\frac{M_L}{300GeV}\right) \left(\frac{(d_e)_{exp}}{1.05\times 10^{-27} ecm}\right)$

Table 4: Experimental constraints for the off diagonal elements of sfermion mass matrices from meson mass splittings $\Delta m_K, \Delta m_B, \Delta m_D$, CP violating parameter ϵ_K, lepton flavor violations $l_i \to l_j \gamma$ and electric dipole moments of neutron d_n and electron d_e. The predictions of our model for each parameters are given in "order" column. The dependences of each upper bounds on experimental values are given in "coefficient" column.

5 Cosmological Aspects

Based on our model, we consider the scenario to reproduce the cosmological parameters given as follows [19]

$$\Omega_0 \quad \simeq \quad \Omega_\Lambda + \Omega_b + \Omega_{CDM} \simeq 1, \tag{187}$$

$$
\begin{aligned}
\Omega_\Lambda &= 0.73 \pm 0.03. && (188)\\
\Omega_b h^2 &= 0.0225 \pm 0.0006, && (189)\\
\Omega_{CDM} h^2 &= 0.112 \pm 0.006, && (190)\\
h &= 0.704 \pm 0.025. && (191)
\end{aligned}
$$

For Ω_b, we adopt leptogenesis as the mechanism to generate baryon asymmetry. For Ω_{CDM}, we assume that dark matter consists of singlino dominated neutralino.

5.1 Leptogenesis

In general, leptogenesis scenario to generate baryon asymmetry causes over production of gravitino in supersymmetric model. This problem can be avoided in the case neutrino mass is generated by small VEV of neutrinophilic Higgs doublet [23].

In the diagonal RHN mass basis, superpotential of RHN is given by

$$
W_N = \sum_{i=1,2} \epsilon^3 H_i^U (L_1, L_2, L_3) \begin{pmatrix} 0 & Y^N_{i,12} & Y^N_{i,13} \\ 0 & Y^N_{i,22} & Y^N_{i,23} \\ 0 & Y^N_{i,32} & Y^N_{i,33} \end{pmatrix} \begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix} + \frac{1}{2}\sum_{a=1}^{3} M_a N_a^c N_a^c, \tag{192}
$$

where we assume accidental mass hierarchy as follows

$$
M_1 = 10^{3.5}, \quad M_2 = M_3 = 10^4 \quad (\mathrm{GeV}). \tag{193}
$$

Note that these particles are enough light to create in low reheating temperature such as 10^7GeV without causing gravitino over production [9]. The interactions of right-handed sneutrinos (RHsNs) are given by

$$
\begin{aligned}
-\mathcal{L}_N &= \sum_{i=1,2} \epsilon^3 H_i^U (L_1, L_2, L_3) \begin{pmatrix} 0 & Y^N_{i,12} M_2 & Y^N_{i,13} M_3 \\ 0 & Y^N_{i,22} M_2 & Y^N_{i,23} M_3 \\ 0 & Y^N_{i,32} M_2 & Y^N_{i,33} M_3 \end{pmatrix} \begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix} \\
&+ \sum_{i=1,2} \epsilon^3 h_i^U (l_1, l_2, l_3) \begin{pmatrix} 0 & Y^N_{i,12} & Y^N_{i,13} \\ 0 & Y^N_{i,22} & Y^N_{i,23} \\ 0 & Y^N_{i,32} & Y^N_{i,33} \end{pmatrix} \begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix}, \qquad (194)
\end{aligned}
$$

where the contributions from A-terms are neglected. The Z_2^N breaking scalar squared mass terms

$$
K \supset \frac{F^*_{B_+} F_{B_{2-}}}{M_P^2}[(N_1^c)^* N_2^c + \cdots] + h.c. = \epsilon m^2 [(N_1^c)^* N_2^c + \cdots] + h.c \tag{195}
$$

fill in the zeros of sneurino mass matrix and gives

$$
\begin{pmatrix} M_1^2 & \epsilon m^2 & \epsilon m^2 \\ \epsilon m^2 & M_2^2 & m^2 \\ \epsilon m^2 & m^2 & M_3^2 \end{pmatrix} \sim M_2^2 \begin{pmatrix} \epsilon & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \tag{196}
$$

where $O(\epsilon)$ suppressions of Z_2^N breaking terms are assumed without any reason. Note that the $O(\epsilon^3)$ elements are originated from small Z_2^N breaking parameters and small Y^Φ as

$$
\frac{m^2 \epsilon}{m^2}\left(\frac{m}{M_2}\right)^2 \sim \epsilon (Y^\Phi)^2 \sim \epsilon^3. \tag{197}
$$

In the diagonal RHsN mass basis, the interaction terms given in Eq.(194) are rewritten by

$$
\begin{aligned}
-\mathcal{L}_N &= \sum_{i=1,2} \epsilon^3 H_i^U (L_1, L_2, L_3) \begin{pmatrix} \epsilon^3 Y^N_{i,11} & Y^N_{i,12} M_2 & Y^N_{i,13} M_3 \\ \epsilon^3 Y^N_{i,21} & Y^N_{i,22} M_2 & Y^N_{i,23} M_3 \\ \epsilon^3 Y^N_{i,31} & Y^N_{i,32} M_2 & Y^N_{i,33} M_3 \end{pmatrix} \begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix} \\
&+ \sum_{i=1,2} \epsilon^3 h_i^U (l_1, l_2, l_3) \begin{pmatrix} \epsilon^3 Y^N_{i,11} & Y^N_{i,12} & Y^N_{i,13} \\ \epsilon^3 Y^N_{i,21} & Y^N_{i,22} & Y^N_{i,23} \\ \epsilon^3 Y^N_{i,31} & Y^N_{i,32} & Y^N_{i,33} \end{pmatrix} \begin{pmatrix} N_1^c \\ N_2^c \\ N_3^c \end{pmatrix}. \qquad (198)
\end{aligned}
$$

The lightest RHN n_1^c does not receive above corrections and remains decoupled. Therefore lepton asymmetry is generated by the out of equilibrium decay of the lightest RHsN N_1^c.

Following [24], the CP asymmetry of sneutrino N_1^c decay is calculated as follows

$$\epsilon_1 = -\frac{1}{4\pi}\sum_k \frac{\mathrm{Im}[K_{1k}^2]}{K_{11}} g(x_k), \tag{199}$$

$$g(x) = \sqrt{x}\ln\frac{1+x}{x} + \frac{2\sqrt{x}}{x-1}, \tag{200}$$

$$x_k = \frac{M_k^2}{M_1^2}, \tag{201}$$

$$K_{ij} = \sum_{h=1,2}\sum_{l=1}^{3}(Y_{h,li}^N)(Y_{h,lj}^N)^*. \tag{202}$$

From the naive power counting, we obtain

$$K_{11} \sim \epsilon^{12}, \quad K_{12} \sim K_{13} \sim \epsilon^9, \quad \epsilon_1 \sim \epsilon^6. \tag{203}$$

Using ϵ_1, the $B-L$ asymmetry generated via thermal leptogenesis is expressed as

$$-(B-L)_f = \kappa\frac{\epsilon_1}{g_*}, \quad g_* = 341.25, \tag{204}$$

where g_* is the total number of relativistic degrees of freedom contributing to the energy density of the universe and dilution factor κ is defined as follows

$$\kappa \sim \frac{O(0.1)}{K}, \tag{205}$$

$$K = \frac{\Gamma(M_1)}{2H(M_1)}, \tag{206}$$

$$\Gamma(M_1) = \frac{K_{11}M_1}{8\pi}, \tag{207}$$

$$H(M_1) = \sqrt{\frac{\pi^2 g_* M_1^4}{90 M_P^2}}. \tag{208}$$

By the EW sphaleron processes, the $B-L$ asymmetry is transferred to a B asymmetry as

$$B_f = \frac{24+4N_H}{66+13N_H}(B-L)_f \sim \frac{1}{3}(B-L)_f, \tag{209}$$

where N_H is number of Higgs doublets which are in equilibrium through Yukawa interactions, for example $N_H = 1$ for SM and $N_H = 2$ for MSSM. In any way N_H-dependence is not important for our rough estimation. For our parameter values, we obtain $K \sim O(1)$ and

$$B_f \sim 10^{-10}, \tag{210}$$

which is consistent with observed value

$$\eta_B = 7.04 B_f = 6.1 \times 10^{-10}. \tag{211}$$

Requiring the effective interaction

$$\mathcal{L}_{\mathrm{eff}} = \epsilon^6 \frac{(H_i^U L_j)^2}{2M_2} \tag{212}$$

is decoupled in order to avoid too strong wash out, we impose the condition as follows

$$\Gamma \sim \frac{\epsilon^{12}T^3}{8\pi^3 M_2^2} < H = \sqrt{\frac{\pi^2 g_* T^4}{90 M_P^2}}, \tag{213}$$

which gives upper bound for temperature as

$$T < 10^4 \mathrm{GeV}. \tag{214}$$

This condition is always satisfied after the decay of N_1^c starts.

This scenario is different from conventional one in the point that neutrino mass

$$m_\nu \sim \frac{10^{-6} v_u^2}{M_2} \sim \left(\frac{v_u}{\mathrm{GeV}}\right)^2 0.1\mathrm{eV} \sim O(0.01\mathrm{eV}), \tag{215}$$

is realized by the small VEV $v_u = O(1\mathrm{GeV})$.

5.2 Dark matter

Here we calculate the relic abundance of LSP which corresponds to singlino dominated neutralino in our model [25]. The most dominant contribution to annihilation cross section of LSP is given by the interaction with Z boson. If the mass matrix given in Eq.(97) is diagonalized by the field redefinition as

$$\begin{pmatrix} (h_i^U)^0 \\ (h_i^D)^0 \\ s_i \end{pmatrix} = \begin{pmatrix} V_a & * & * \\ V_b & * & * \\ V_c & * & * \end{pmatrix} \begin{pmatrix} \chi_{i,1}^0 \\ \chi_{i,2}^0 \\ \chi_{i,3}^0 \end{pmatrix}, \quad m_{\chi_{i,1}^0} < m_{\chi_{i,2}^0} < m_{\chi_{i,3}^0}, \quad (i=1,2), \tag{216}$$

the interaction with Z boson is given by

$$\begin{aligned}
\mathcal{L}_Z &= G(\chi_{1,1}^0)\bar{\chi}_{i,1}^0 Z_\mu \bar{\sigma}^\mu \chi_{i,1}^0 + iG(f_L)\bar{f}\gamma^\mu Z_\mu P_L f + iG(f_R)\bar{f}\gamma^\mu Z_\mu P_R f, && (217)\\
& G(\chi_{1,1}^0) = \frac{1}{2}(V_a^2 - V_b^2)\sqrt{g_Y^2 + g_2^2} = 0.372(|V_a|^2 - |V_b|^2) && (218)\\
& G(e_L) = 0.200, \quad G(e_R) = -0.172, \quad G(\nu_L) = -0.372, && \\
& G(u_L) = -0.257, \quad G(u_R) = 0.115, \quad G(d_L) = 0.314, \quad G(d_R) = -0.057, && (219)
\end{aligned}$$

where

$$\alpha_Y(m_Z) = 0.0101687, \quad \alpha_2(m_Z) = 0.0338098 \tag{220}$$

are used.

The formula for the relic abundance of cold dark matter is given by

$$\begin{aligned}
\Omega_{CDM} h^2 &= \frac{8.76 \times 10^{-11} g_*^{-\frac{1}{2}} x_F}{(a + 3b/x_F)\mathrm{GeV}^2}, && (221)\\
x_F &= \ln \frac{0.0955 m_P m_{\chi_1^0}(a + 6b/x_F)}{(g_* x_F)^{\frac{1}{2}}}, && (222)\\
& m_P = 1.22 \times 10^{19}\mathrm{GeV}, && \\
& g_* = 72.25 \quad (T_F = m_{\chi_1^0}/x_F < m_\tau), && (223)\\
a &= \sum_f \frac{c_f}{2\pi} G^2(\chi_{1,1}^0) \left[\frac{m_f}{4m_{\chi_1^0}^2 - m_Z^2}[G(f_L) - G(f_R)]\right]^2 && (224)\\
b &= \sum_f \frac{c_f}{3\pi} G^2(\chi_{1,1}^0) \left(\frac{m_{\chi_1^0}}{4m_{\chi_{1,1}^0}^2 - m_Z^2}\right)^2 [G^2(f_L) + G^2(f_R)] - \frac{3}{4}a && (225)
\end{aligned}$$

where $m_f \ll m_{\chi^0}$ is assumed. Substituting the values given in Eq.(218) and Eq.(219) and following values

$$m_Z = 91.1876, \quad m_b = 4.18, \quad m_\tau = 1.777 \quad (\mathrm{GeV}), \quad r = \frac{2m_{\chi_{1,1}^0}}{m_Z} \tag{226}$$

in Eq.(224) and Eq.(225), we get

$$\begin{aligned}
a &= 1.77 \times 10^{-8} \left(\frac{G(\chi_{1,1}^0)}{r^2 - 1}\right)^2 (\mathrm{GeV}^{-2}), && (227)\\
b &= 6.436 \times 10^{-6} \left(\frac{G(\chi_{1,1}^0) r}{r^2 - 1}\right)^2 - 0.013 \times 10^{-6} \left(\frac{G(\chi_{1,1}^0)}{r^2 - 1}\right)^2 (\mathrm{GeV}^{-2}), && (228)
\end{aligned}$$

from which the formula is rewritten as follows

$$x_F = \ln\left[\left(\frac{G(\chi^0_{1,1})}{0.01}\right)^2\left(0.0177+\frac{6}{x_F}(6.436r^2-0.013)\right)\frac{6.25\times10^8 r}{(r^2-1)^2}\right]-\frac{1}{2}\ln(x_F), \tag{229}$$

$$\Omega_{CDM}h^2 = \frac{0.10306x_F}{\left(\frac{G(\chi^0_{1,1})}{0.01}\right)^2\left(0.0177+\frac{3}{x_F}(6.436r^2-0.013)\right)\frac{1}{(r^2-1)^2}}, \tag{230}$$

where quark and lepton masses are neglected except for bottom and τ. Since the two LSPs $\chi^0_{1,1}, \chi^0_{2,1}$ have the same mass and the same interactions, they have the same relic abundance. Therefore the required relic abundance of one LSP is $\Omega_{CDM}h^2 = 0.055$. For the allowed range given in Eq.(104), the required values for $\lambda_{4,5}$ to reproduce observed relic abundance of dark matter are given in Table 5. The allowed ranges for $\lambda_{4,5}$ are very small. Note that we should not impose LEP bound ($m_{\chi^0_1} > 46$GeV) on this LSP, because $Z \to \chi^0_{i,1}\chi^0_{i,1}$ is strongly suppressed by the factor $(|V_a|^2-|V_b|^2)^2/2 \sim 0.005$ and the contribution to invisible decay width is negligible as follows

$$\Gamma(Z\to\chi^0_{i,1}\chi^0_{i,1}) \sim (0.6\times 2/3)0.005\Gamma(\text{invisible}) \sim 1.0\text{MeV}, \tag{231}$$

$$\Gamma(\text{invisible}) = 499.0 \pm 1.5\text{MeV}[19], \tag{232}$$

where phase space suppression factor ~ 0.6 and the ratio of LSP number and neutrino number $2/3$ are multiplied.

λ_4	λ_5	$m_{\chi^\pm_1}$	$m_{\chi^0_{1,1}}$	$m_{\chi^0_{1,2}}$	$m_{\chi^0_{1,3}}$	V_a	V_b	x_F	T_F	$\Omega_{CDM}h^2$
0.44	0.57	141.2	36.47	142.38	178.84	0.3667	0.2225	22.90	1.592	0.0552
0.42	0.56	130.1	36.52	131.09	167.61	0.3721	0.2321	22.90	1.595	0.0550
0.40	0.55	119.1	36.61	119.91	156.52	0.3776	0.2429	22.90	1.599	0.0551
0.38	0.54	108.0	36.79	108.63	145.42	0.3836	0.2554	22.91	1.606	0.0549
0.37	0.53	102.5	36.60	103.10	139.69	0.3885	0.2591	22.91	1.597	0.0550

Table 5: The parameter sets $(\lambda_4, \lambda_5, m^\pm_{\chi_1} = \lambda_1 v'_s)$ which reproduce observed relic abundance of dark matter. The dimensionful values are expressed in GeV units.

5.3 Constraint for long-lived massive particles

Finally we consider long-lived massive particles which are included in our model, G-Higgs, flavons and the lightest RHN. Such particles are imposed on strong constraints from cosmological observations.

The superpotential of G-Higgs sector gives degenerated G-higgsino mass as

$$M_g = kv'_s\text{diag}(1,1,1), \tag{233}$$

which receives S_4 breaking perturbation from Kähler potential given by

$$\begin{aligned} K(G) &= |G_a|^2 + \frac{1}{M_P^2}\left[|2D_2G_1|^2 + |(-\sqrt{3}D_1-D_2)G_2|^2 + |(\sqrt{3}D_1-D_2)G_3|^2\right] + (G\to G^c) \\ &= |G_a|^2 + \sum_a c_a\epsilon^2|G_a|^2 + (G\to G^c), \end{aligned} \tag{234}$$

which solves the mass degeneracy, however generation mixing is not induced. Neglecting $O(\epsilon^2)$ corrections and contributions from D-terms except for the contribution from S_3, the G-Higgs mass terms are given by

$$\begin{aligned} -\mathcal{L} &\supset m_G^2|G_a|^2 + m^2_{G^c}|G^c_a|^2 - [kA_kS_3G_aG^c_a + h.c.] + |kS_3G_a|^2 + |kS_3G^c_a|^2 \\ &+ \left|kG_aG^c_a + \lambda_3H^U_3H^D_3\right|^2 + \frac{1}{2}g_x^2\left[5|S_3|^2 - 2|G_a|^2 - 3|G^c_a|^2\right]^2, \end{aligned} \tag{235}$$

from which we obtain three same 2×2 matrices as

$$M^2_a(G) = \begin{pmatrix} m_G^2 + (kv'_s)^2 - 10g_x^2(v'_s)^2 & \lambda_3kv'_uv'_d - kA_kv'_s \\ \lambda_3kv'_uv'_d - kA_kv'_s & m^2_{G^c} + (kv'_s)^2 - 15g_x^2(v'_s)^2 \end{pmatrix}. \tag{236}$$

The mass spectrum of G-Higgs and G-higgsino is given in Table 7 and the lightest particle of them is lighter G-Higgs scalar G_-. The dominant contributions to the G_- decay are given by the superpotential

$$\begin{aligned} W &\supset \frac{1}{M_P^2} Q_3 Q_3 \Phi_3^c \sum_a \Phi_a G_a + \frac{1}{M_P^2} U_3^c E_3^c \Phi_3^c \sum_a \Phi_a G_a \\ &= \frac{1}{\sqrt{3}} Y^{QQ} Q_3 Q_3 (G_1 + G_2 + G_3) + \frac{1}{\sqrt{3}} Y^{UE} U_3^c E_3^c (G_1 + G_2 + G_3), \qquad (237) \\ & \quad Y^{QQ} = Y^{UE} = \left(\frac{\langle \Phi_3 \rangle}{M_P} \right)^2 \sim 2 \times 10^{-14}, \qquad (238) \end{aligned}$$

from which we obtain

$$\mathcal{L}_G = \frac{1}{\sqrt{3}} A_{RF}^{UE} Y^{UE} (e_3^c u_3^c + u_3^c e_3^c) G_1 + \frac{1}{\sqrt{3}} A_{RF}^{QQ} Y^{QQ} (2u_3 d_3 + 2d_3 u_3) G_1. \qquad (239)$$

For simplicity, we assume $G \sim G_-$ then decay width of G_- is given by

$$\Gamma(G_-) = \frac{M(G_-)}{16\pi} \left[2 \left(\frac{1}{\sqrt{3}} A_{RF}^{UE} \right)^2 + 4 \left(\frac{2}{\sqrt{3}} A_{RF}^{QQ} \right)^2 \right] (Y^{QQ})^2, \qquad (240)$$

where the renormalization factors are calculated based on the RGEs given in appendix A as follows

$$A_{RF}^{UE} = \sqrt{\frac{\alpha_{UE}(M_S)}{\alpha_{UE}(M_P)}} = 4.9, \quad A_{RF}^{QQ} = \sqrt{\frac{\alpha_{QQ}(M_S)}{\alpha_{QQ}(M_P)}} = 12.8. \qquad (241)$$

Substituting these values in Eq.(240) we obtain the life time of G_- as

$$\tau(G_-) = \frac{1}{\Gamma(G_-)} = 3.8 \times 10^{-29} \left(\frac{M(G_-)}{1\text{TeV}} \right)^{-1} (Y^{QQ})^{-2} \quad \text{sec.} \qquad (242)$$

Since the existence of a particle which has longer life time than 0.1 second spoils the success of BBN [9], we must require $\tau(G_-) < 0.1$sec which impose constraint as

$$M(G_-) > \left(\frac{1.9 \times 10^{-14}}{Y^{QQ}} \right)^2 \text{TeV}. \qquad (243)$$

The G-Higgs exchange may contribute to proton decay, however it seems that the suppression of power of ϵ is too strong to observe proton decay [26].

The five of six flavon multiplets Φ_a, Φ_a^c have $O(1\text{TeV})$ masses which are enough small to product them non-thermally through the $U(1)_Z$ gauge interaction. The lightest flavon (LF) is quasi-stable and should not produced so much in order not to dominate Ω_{CDM}. Solving the Boltzmann equation with the boundary condition $n_{LF}(T_{RH}) = 0$, we get relic abundance of LF as [1]

$$\Omega_{LF} h^2 = 2.0 \times 10^{-8} \left(\frac{T_{RH}}{10^5\text{GeV}} \right)^3 \left(\frac{10^{12}\text{GeV}}{\langle \Phi_3^c \rangle} \right)^4 = 2.0 \times 10^{-6} \left(\frac{T_{RH}}{10^5\text{GeV}} \right)^3 [27]. \qquad (244)$$

Requiring the LF does not dominate dark matter as $\Omega_{LF} h^2 < 0.01$, the upper bound for reheating temperature is given by

$$T_R < 10^6\text{GeV}, \qquad (245)$$

which is consistent with our leptogenesis scenario.

The life time of LF is estimated as follows. The LF can decay, for example through the operator

$$W = \frac{M_P (\epsilon^3 H_i^U L_j)^2}{2(V + \Phi)^2} \sim \frac{M_P (\epsilon^3 H_i^U L_j)^2}{2V^2} \left(1 - 2\frac{\Phi}{V} \right), \qquad (246)$$

[1]Since the $U(1)_Z$ charge of Φ in [27] is two times larger than one in the present model, we multiply the equation for $\Omega_{LF} h^2$ given in [27] by the factor 2^2.

the decay width and life time are given by

$$\Gamma(LF \to llHH) = \frac{M_S}{16\pi}\left(\frac{M_S^2(\epsilon^3)^2}{32\pi^2 M_2 V}\right)^2 O(0.1) \sim 10^{-29}\text{eV}, \tag{247}$$

$$\tau(LF) \sim 10^{14}\text{sec} \sim 10^7\text{years}[27], \tag{248}$$

which suggests LF does not exist in present universe. Note that three and two body decays are suppressed by small VEV v_u.

The lightest RHN n_1^c behaves like LF because there is no distinction between N^c and Φ^c under the gauge symmetry. Integrating out N_1^c and λ_Z in the Lagrangian

$$\mathcal{L} \supset g_Z\left(n_1^c \lambda_Z (N_1^c)^* + \psi \lambda_Z \Psi^*\right) + \epsilon^6 N_1^c l h_i^U, \tag{249}$$

where (ψ, Ψ) means super-multiplet and some factors are omitted for simplicity, we get

$$\mathcal{L}_{\text{eff}} = \frac{g_Z^2(\epsilon^6)}{(g_Z V) M_1^2}(n_1^c \psi)(l h_i^U)\Psi, \tag{250}$$

from which the life time of n_1^c is given by

$$\Gamma(n_1^c \to \psi \Psi l h^U) = \frac{M_1^7}{16\pi(32\pi^2)^2}\left(\frac{g_Z^2(\epsilon^6)}{(g_Z V)M_1^2}\right)^2 \sim \left(\frac{M_1}{M_S}\right)^5 \Gamma(LF \to llHH) \sim 10^{-22}\text{eV}, \tag{251}$$

$$\tau(n_1^c) \sim \left(\frac{M_S}{M_1}\right)^5 \tau(LF) \sim 10^{12}\text{sec} \sim 10^5\text{years}. \tag{252}$$

6 Conclusion

In this paper we consider S_4 flavor symmetric extra U(1) model and obtain following results.

- With the assignment of flavor representation to reproduce quark and lepton mass hierarchies and mixing matrices, SUSY flavor problem is softened.
- Proton decay through G-Higgs exchange is suppressed by flavor symmetry.
- Observed Higgs mass $125-126$GeV is realized with stop lighter than 2TeV which is within the testable range in LHC at $\sqrt{s} = 14$TeV.
- The partial gauge coupling unification at M_P is realized by adding 4-th generation Higgs and left-handed lepton which play the role to break $U(1)_Z$ gauge symmetry.
- The allowed region for lightest chargino mass is given by $100-140$GeV when we assume LSP is lightest singlino dominated neutralino.
- The extra Higgs doublets play the role of neutrinophilic Higgs which is needed for low temperature leptogengesis without causing gravitino over production.
- The shorter life time than 0.1 second of G-Higgs is realized.
- The over productions of flavon and lightest RHN are also avoided.

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A RGEs

$O(1)$ coupling constants of our model consist of gauge coupling constants and trilinear coupling constants defined by

$$\begin{aligned} W &\supset \lambda_3 S_3 H_3^U H_3^D + \lambda_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 (S_1 H_1^U + S_2 H_2^U) H_3^D \\ &+ k S_3 (G_1 G_1^c + G_2 G_2^c + G_3 G_3^c) + Y_3^U H_3^U Q_3 U_3^c, \end{aligned} \tag{253}$$

from which the fine structure constants are defined as follows

$$\begin{aligned} &\alpha_Y = \frac{g_Y^2}{4\pi}, \quad \alpha_2 = \frac{g_2^2}{4\pi}, \quad \alpha_3 = \frac{g_3^2}{4\pi}, \quad \alpha_X = \frac{g_X^2}{4\pi}, \quad \alpha_Z = \frac{g_Z^2}{4\pi}, \\ &\alpha_t = \frac{(Y_3^U)^2}{4\pi}, \quad \alpha_h = \frac{\lambda_3^2}{4\pi}, \quad \alpha_4 = \frac{\lambda_4^2}{4\pi}, \quad \alpha_5 = \frac{\lambda_5^2}{4\pi}, \quad \alpha_k = \frac{k^2}{4\pi}. \end{aligned} \tag{254}$$

We define the step functions as follows

$$\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}, \tag{255}$$

$$\begin{aligned} &\theta_I = \theta(\mu - M_I), \quad \theta_4 = \theta(\mu - M_{L_4}), \quad \theta_5 = \theta(\mu - M_{L_5}), \\ &M_I = 10^{11.5}\text{GeV}, \quad M_{L_4} = 2.2 \times 10^{14}\text{GeV}, \quad M_{L_5} = 2.4 \times 10^{17}\text{GeV}. \end{aligned} \tag{256}$$

The beta functions are given by

$$(2\pi)\frac{d\alpha_Y}{dt} = \alpha_Y^2 \left[15 + 20\frac{\alpha_3}{2\pi} + \frac{15}{2}\frac{\alpha_2}{2\pi} + \left(2 + 3\frac{\alpha_2}{2\pi}\right)\theta_4 + \left(\frac{10}{3} + 3\frac{\alpha_2}{2\pi} + \frac{32}{9}\frac{\alpha_3}{2\pi}\right)\theta_5\right], \tag{257}$$

$$(2\pi)\frac{d\alpha_2}{dt} = \alpha_2^2 \left[3 + 12\frac{\alpha_3}{2\pi} + \frac{39}{2}\frac{\alpha_2}{2\pi} + \left(2 + 7\frac{\alpha_2}{2\pi}\right)\theta_4 + \left(2 + 7\frac{\alpha_2}{2\pi}\right)\theta_5\right], \tag{258}$$

$$(2\pi)\frac{d\alpha_3}{dt} = \alpha_3^2 \left[24\frac{\alpha_3}{2\pi} + \frac{9}{2}\frac{\alpha_2}{2\pi} + \left(2 + \frac{34}{3}\frac{\alpha_3}{2\pi}\right)\theta_5\right], \tag{259}$$

$$(2\pi)\frac{d\alpha_X}{dt} = \alpha_X^2 \left[15 + 20\frac{\alpha_3}{2\pi} + \frac{15}{2}\frac{\alpha_2}{2\pi} + \left(\frac{4}{3} + 2\frac{\alpha_2}{2\pi}\right)\theta_4 + \left(\frac{10}{3} + 2\frac{\alpha_2}{2\pi} + \frac{16}{3}\frac{\alpha_3}{2\pi}\right)\theta_5\right], \tag{260}$$

$$(2\pi)\frac{d\alpha_Z}{dt} = \alpha_Z^2 \left[\frac{65}{3} + 20\frac{\alpha_3}{2\pi} + \frac{15}{2}\frac{\alpha_2}{2\pi} + \left(\frac{20}{9} + \frac{10}{3}\frac{\alpha_2}{2\pi}\right)\theta_4 + \left(\frac{50}{9} + \frac{10}{3}\frac{\alpha_2}{2\pi} + \frac{80}{9}\frac{\alpha_3}{2\pi}\right)\theta_5\right]\theta_I, \tag{261}$$

$$t = \ln \mu, \tag{262}$$

where we include only the contributions from $\alpha_{2,3}$ in 2-loop order terms. The RGEs for trilinear coupling constants are given by

$$(2\pi)\frac{d\alpha_t}{dt} = \alpha_t \left(6\alpha_t + \alpha_h + 2\alpha_4 - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{9}\alpha_Y - \frac{1}{2}\alpha_X - \frac{5}{6}\alpha_Z\theta_I\right), \tag{263}$$

$$(2\pi)\frac{d\alpha_h}{dt} = \alpha_h \left(3\alpha_t + 4\alpha_h + 2\alpha_4 + 2\alpha_5 + 9\alpha_k - 3\alpha_2 - \alpha_Y - \frac{19}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I\right), \tag{264}$$

$$(2\pi)\frac{d\alpha_4}{dt} = \alpha_4 \left(3\alpha_t + \alpha_h + 5\alpha_4 + 2\alpha_5 - 3\alpha_2 - \alpha_Y - \frac{19}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I\right), \tag{265}$$

$$(2\pi)\frac{d\alpha_5}{dt} = \alpha_5 \left(\alpha_h + 2\alpha_4 + 5\alpha_5 - 3\alpha_2 - \alpha_Y - \frac{19}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I\right), \tag{266}$$

$$(2\pi)\frac{d\alpha_k}{dt} = \alpha_k \left(2\alpha_h + 11\alpha_k - \frac{16}{3}\alpha_3 - \frac{4}{9}\alpha_Y - \frac{19}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I\right). \tag{267}$$

We define gaugino mass parameters and A-parameters as follows

$$\begin{aligned} \mathcal{L} &\supset -\frac{1}{2}M_Y\lambda_Y\lambda_Y - \frac{1}{2}M_2\lambda_2\lambda_2 - \frac{1}{2}M_3\lambda_3^g\lambda_3^g - \frac{1}{2}M_X\lambda_X\lambda_X - \frac{1}{2}M_Z\lambda_Z\lambda_Z \\ &+ \lambda_3 A_3 S_3 H_3^U H_3^D + \lambda_4 A_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 A_5 (S_1 H_1^U + S_2 H_2^U) H_3^D \\ &+ k A_k S_3 (G_1 G_1^c + G_2 G_2^c + G_3 G_3^c) + Y_3^U A_t H_3^U Q_3 U_3^c + h.c.. \end{aligned} \tag{268}$$

The RGEs for gaugino mass parameters are given by

$$
\begin{aligned}
(2\pi)\frac{dM_Y}{dt} &= \alpha_Y M_Y \left[15 + 2\theta_4 + \frac{10}{3}\theta_5\right], && (269)\\
(2\pi)\frac{dM_2}{dt} &= \alpha_2 M_2 \left[3 + 2\theta_4 + 2\theta_5\right], && (270)\\
(2\pi)\frac{dM_3}{dt} &= \alpha_3 M_3 \left[2\theta_5 + \frac{48}{2\pi}\alpha_3\right], && (271)\\
(2\pi)\frac{dM_X}{dt} &= \alpha_X M_X \left[15 + \frac{4}{3}\theta_4 + \frac{10}{3}\theta_5\right], && (272)\\
(2\pi)\frac{dM_Z}{dt} &= \alpha_Z M_Z \left[\frac{65}{3} + \frac{20}{9}\theta_4 + \frac{50}{9}\theta_5\right]\theta_I, && (273)
\end{aligned}
$$

where we take account of 2-loop contributions only for M_3. The RGEs for A-parameters are given by

$$
\begin{aligned}
(2\pi)\frac{dA_t}{dt} &= 6\alpha_t A_t + \alpha_h A_3 + 2\alpha_4 A_4 + \frac{16}{3}\alpha_3 M_3 + 3\alpha_2 M_2 + \frac{13}{9}\alpha_Y M_Y \\
&+ \frac{1}{2}\alpha_X M_X + \frac{5}{6}\alpha_Z M_Z \theta_I, && (274)\\
(2\pi)\frac{dA_3}{dt} &= 3\alpha_t A_t + 4\alpha_h A_3 + 2\alpha_4 A_4 + 2\alpha_5 A_5 + 9\alpha_k A_k \\
&+ 3\alpha_2 M_2 + \alpha_Y M_Y + \frac{19}{6}\alpha_X M_X + \frac{5}{6}\alpha_Z M_Z \theta_I, && (275)\\
(2\pi)\frac{dA_4}{dt} &= 3\alpha_t A_t + \alpha_h A_3 + 5\alpha_4 A_4 + 2\alpha_5 A_5 + 3\alpha_2 M_2 + \alpha_Y M_Y + \frac{19}{6}\alpha_X M_X + \frac{5}{6}\alpha_Z M_Z \theta_I, && (276)\\
(2\pi)\frac{dA_5}{dt} &= \alpha_h A_3 + 2\alpha_4 A_4 + 5\alpha_5 A_5 + 3\alpha_2 M_2 + \alpha_Y M_Y + \frac{19}{6}\alpha_X M_X + \frac{5}{6}\alpha_Z M_Z \theta_I, && (277)\\
(2\pi)\frac{dA_k}{dt} &= 2\alpha_h A_3 + 11\alpha_k A_k + \frac{16}{3}\alpha_3 M_3 + \frac{4}{9}\alpha_Y M_Y + \frac{19}{6}\alpha_X M_X + \frac{5}{6}\alpha_Z M_Z \theta_I. && (278)
\end{aligned}
$$

RGEs for scalar squared masses are given by

$$
\begin{aligned}
(2\pi)\frac{dm^2_{Q_a}}{dt} &= \alpha_t M_t^2 \delta_{a,3} - \frac{16}{3}\alpha_3 M_3^2 - 3\alpha_2 M_2^2 - \frac{1}{9}\alpha_Y M_Y^2 - \frac{1}{6}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2 \theta_I, && (279)\\
(2\pi)\frac{dm^2_{U^c_a}}{dt} &= 2\alpha_t M_t^2 \delta_{a,3} - \frac{16}{3}\alpha_3 M_3^2 - \frac{16}{9}\alpha_Y M_Y^2 - \frac{1}{6}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2 \theta_I, && (280)\\
(2\pi)\frac{dm^2_{D^c_a}}{dt} &= -\frac{16}{3}\alpha_3 M_3^2 - \frac{4}{9}\alpha_Y M_Y^2 - \frac{2}{3}\alpha_X M_X^2 - \frac{10}{9}\alpha_Z M_Z^2 \theta_I, && (281)\\
(2\pi)\frac{dm^2_{L_a}}{dt} &= -3\alpha_2 M_2^2 - \alpha_Y M_Y^2 - \frac{2}{3}\alpha_X M_X^2 - \frac{10}{9}\alpha_Z M_Z^2 \theta_I, && (282)\\
(2\pi)\frac{dm^2_{E^c_a}}{dt} &= -4\alpha_Y M_Y^2 - \frac{1}{6}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2 \theta_I, && (283)\\
(2\pi)\frac{dm^2_{H^U_a}}{dt} &= (3\alpha_t M_t^2 + \alpha_h M_h^2 + 2\alpha_4 M_4^2)\delta_{a,3} + \alpha_5 M_5^2 (1 - \delta_{a,3}) \\
&- 3\alpha_2 M_2^2 - \alpha_Y M_Y^2 - \frac{2}{3}\alpha_X M_X^2 - \frac{10}{9}\alpha_Z M_Z^2 \theta_I, && (284)\\
(2\pi)\frac{dm^2_{H^D_a}}{dt} &= \alpha_h M_h^2 \delta_{a,3} + \alpha_4 M_4^2 (1 - \delta_{a,3}) + 2\alpha_5 M_5^2 \delta_{a,3} \\
&- 3\alpha_2 M_2^2 - \alpha_Y M_Y^2 - \frac{3}{2}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2 \theta_I, && (285)\\
(2\pi)\frac{dm^2_{S_a}}{dt} &= (2\alpha_h M_h^2 + 9\alpha_k M_k^2)\delta_{a,3} + (2\alpha_4 M_4^2 + 2\alpha_5 M_5^2)(1 - \delta_{a,3}) \\
&- \frac{25}{6}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2 \theta_I, && (286)\\
(2\pi)\frac{dm^2_{G_a}}{dt} &= \alpha_k M_k^2 - \frac{16}{3}\alpha_3 M_3^2 - \frac{4}{9}\alpha_Y M_Y^2 - \frac{2}{3}\alpha_X M_X^2 - \frac{10}{9}\alpha_Z M_Z^2 \theta_I, && (287)
\end{aligned}
$$

$$(2\pi)\frac{dm^2_{G^c_a}}{dt} = \alpha_k M_k^2 - \frac{16}{3}\alpha_3 M_3^2 - \frac{4}{9}\alpha_Y M_Y^2 - \frac{3}{2}\alpha_X M_X^2 - \frac{5}{18}\alpha_Z M_Z^2\theta_I, \tag{288}$$

where

$$\begin{aligned} M_t^2 &= A_t^2 + m^2_{Q_3} + m^2_{U^c_3} + m^2_{H^U_3}, \quad M_h^2 = A_3^2 + m^2_{S_3} + m^2_{H^U_3} + m^2_{H^D_3}, \\ M_4^2 &= A_4^2 + m^2_{S_1} + m^2_{H^U_3} + m^2_{H^D_1}, \quad M_5^2 = A_5^2 + m^2_{S_1} + m^2_{H^U_1} + m^2_{H^D_3}, \\ M_k^2 &= A_k^2 + m^2_{S_3} + m^2_{G_1} + m^2_{G^c_1}. \end{aligned} \tag{289}$$

Note that the relations

$$\begin{aligned} &m^2_{S_1} = m^2_{S_2} = m^2_S, \quad m^2_{H^U_1} = m^2_{H^U_2} = m^2_{H^U}, \quad m^2_{H^D_1} = m^2_{H^D_2} = m^2_{H^D}, \quad m^2_{Q_1} = m^2_{Q_2} = m^2_Q, \\ &m^2_{L_1} = m^2_{L_2} = m^2_L, \quad m^2_{G_1} = m^2_{G_2} = m^2_{G_3} = m^2_G, \quad m^2_{G^c_1} = m^2_{G^c_2} = m^2_{G^c_3} = m^2_{G^c}, \end{aligned} \tag{290}$$

are held. At $\mu = M_I$, we add $U(1)_Z$ D-term corrections as follows [28]

$$m_X^2(M_I - 0) = m_X^2(M_I + 0) + \Delta m_X^2 \tag{291}$$

$$\begin{aligned} &\Delta m_Q^2 = \Delta m^2_{U^c} = \Delta m^2_{E^c} = \Delta m^2_{H^D} = \Delta m^2_{G^c} = \Delta m^2_S = \frac{5}{18} m^2_{DT}, \\ &\Delta m^2_{D^c} = \Delta m^2_L = \Delta m^2_{H^U} = \Delta m^2_G = -\frac{5}{9} m^2_{DT}, \end{aligned} \tag{292}$$

$$m^2_{DT} = 1\text{TeV}^2 > 0. \tag{293}$$

We solve these RGEs using following boundary conditions. At SUSY breaking scale ($\mu = M_S = 1$TeV), we put by hand as follows

$$\begin{aligned} &\lambda_3 = 0.37, \quad \lambda_4 = 0.4, \quad \lambda_5 = 0.55, \quad Y_t = Y^U_3 = 1.0, \quad k = 0.5, \quad M_3 = 1000\text{GeV}, \quad M_Y = 200\text{GeV}, \\ &m^2_{Q_3} = 3.00, \quad m^2_{U^c_3} = 1.00, \quad m^2_{H^U} = m^2_{H^D} = m^2_S = 2.00, \quad m^2_G = 5.50, \quad m^2_{G^c} = 7.00 \quad (\text{TeV}^2), \\ &m^2_{H^U_3, H^D_3, S_3} \to \text{Eq.}(67)(68)(69). \end{aligned} \tag{294}$$

At reduced Planck scale ($\mu = M_P = 2.4 \times 10^{18}$GeV),we put by hand as follows

$$\begin{aligned} &\alpha_2 = \alpha_3 = 0.125, \quad \alpha_X = \alpha_Z = \alpha_Y = 0.209, \quad M_2 = M_3, \quad M_X = M_Y = M_Z, \quad A_{t,3,4,5,k} = 0, \\ &m^2_{L_a} = m^2_{E^c_a} = m^2_{D^c_a} = m^2_{U^c_i} = m^2_Q = 0. \end{aligned} \tag{295}$$

Note that gauge coupling constants do not satisfy the conventional unification as

$$\alpha_Y = \frac{3}{5}\alpha_{2,3}. \tag{296}$$

The renormalization factors of first and second generation Yukawa coupling constants $Y^{u,d,e}$ and single G-Higgs coupling constants defined by

$$W_G = Y^{QQ} Q_3 Q_3 (G_1 + G_2 + G_3) + Y^{UE} U^c_3 E^c_3 (G_1 + G_2 + G_3), \tag{297}$$

are given by

$$\sqrt{\frac{\alpha_A(M_S)}{\alpha_A(M_P)}}, \quad \alpha_A = \frac{|Y^A|^2}{4\pi}, \quad A = u, d, e, QQ, UE, \tag{298}$$

which are calculated by RGEs as follows

$$(2\pi)\frac{1}{\alpha_u}\frac{d\alpha_u}{dt} = 3\alpha_t + \alpha_h + 2\alpha_4 - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{9}\alpha_Y - \frac{1}{2}\alpha_X - \frac{5}{6}\alpha_Z\theta_I, \tag{299}$$

$$(2\pi)\frac{1}{\alpha_d}\frac{d\alpha_d}{dt} = \alpha_h + 2\alpha_5 - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{9}\alpha_Y - \frac{7}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I, \tag{300}$$

$$(2\pi)\frac{1}{\alpha_e}\frac{d\alpha_e}{dt} = \alpha_h + 2\alpha_5 - 3\alpha_2 - 3\alpha_Y - \frac{7}{6}\alpha_X - \frac{5}{6}\alpha_Z\theta_I, \tag{301}$$

$$(2\pi)\frac{1}{\alpha_{UE}}\frac{d\alpha_{UE}}{dt} = 2\alpha_t + \alpha_k - \frac{16}{3}\alpha_3 - \frac{28}{9}\alpha_Y - \frac{1}{2}\alpha_X - \frac{5}{6}\alpha_Z\theta_I, \tag{302}$$

$$(2\pi)\frac{1}{\alpha_{QQ}}\frac{d\alpha_{QQ}}{dt} = 2\alpha_t + \alpha_k - 8\alpha_3 - 3\alpha_2 - \frac{1}{3}\alpha_Y - \frac{1}{2}\alpha_X - \frac{5}{6}\alpha_Z\theta_I, \tag{303}$$

where the contributions from Φ_a, Φ^c_3, D_i are neglected. The results are given in Table 6.

parameter	$\mu = M_S(M_I)$	$\mu = M_P$	parameter	$\mu = M_S$	$\mu = M_P$
α_Y	0.010442	0.209	$m^2_{Q_3}$	3.00	0.9912
α_2	0.032482	0.125	$m^2_{U^c_3}$	1.00	3.1451
α_3	0.089430	0.125	$m^2_{H^U_3}$	-0.1723	16.4770
α_X	0.010552	0.209	$m^2_{H^U}$	2.00	2.4671
α_Z	(0.015162)	0.209	$m^2_{H^D_3}$	2.6811	6.2912
α_t	0.079577	0.006455	$m^2_{H^D}$	2.00	1.6459
α_h	0.010894	0.016086	$m^2_{S_3}$	-2.2105	10.4216
α_4	0.012732	0.011761	m^2_S	2.00	6.4914
α_5	0.024072	0.011309	m^2_G	5.50	1.6852
α_k	0.019894	0.001014	$m^2_{G^c}$	7.00	2.2501
M_Y	0.2	3.68582	m^2_Q	5.9677	0.0
M_2	0.49889	1.68366	$m^2_{U^c_i}$	5.7726	0.0
M_3	1.0	1.68366	$m^2_{D^c_a}$	4.8538	0.0
M_X	0.20228	3.68582	$m^2_{L_a}$	1.0559	0.0
M_Z	(0.28352)	3.68582	$m^2_{E^c_a}$	1.7796	0.0
A_t	-1.78127	0.0	α_u	0.259802	0.01
A_3	1.51978	0.0	α_d	0.523940	0.01
A_4	0.40541	0.0	α_e	0.037691	0.01
A_5	-0.93253	0.0	α_{UE}	0.238230	0.01
A_k	-3.05177	0.0	α_{QQ}	1.638921	0.01

Table 6: Each boundary values of the solutions of RGEs. The dimensionful parameters are expressed in TeV units. The experimental values of gauge coupling constants give $\alpha_Y(M_S) = 0.010445, \alpha_2(M_S) = 0.032484, \alpha_3(M_S) = 0.089514$ which are calculated based on SM RGEs. The values of α_Z, M_Z at low energy side are given by the values at $\mu = M_I$ in brackets.

B The multiplication rules of S_4

The representations of S_4 are $1, 1', 2, 3, 3'$ [29]. Their products are expanded as follows.

$$\begin{aligned}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_3 \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_3 &= (x_1y_1 + x_2y_2 + x_3y_3)_1 + \begin{pmatrix} \sqrt{3}(x_2y_2 - x_3y_3) \\ (x_2y_2 + x_3y_3 - 2x_1y_1) \end{pmatrix}_2 + \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}_{3'} \\
&+ \begin{pmatrix} x_2y_3 + x_3y_2 \\ x_3y_1 + x_1y_3 \\ x_1y_2 + x_2y_1 \end{pmatrix}_3 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3'} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3'}, \qquad (304)
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_3 \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3'} &= (x_1y_1 + x_2y_2 + x_3y_3)_{1'} + \begin{pmatrix} (x_2y_2 + x_3y_3 - 2x_1y_1) \\ -\sqrt{3}(x_2y_2 - x_3y_3) \end{pmatrix}_2 \\
&+ \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}_3 + \begin{pmatrix} x_2y_3 + x_3y_2 \\ x_3y_1 + x_1y_3 \\ x_1y_2 + x_2y_1 \end{pmatrix}_{3'}, \qquad (305)
\end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{3(3')} = \begin{pmatrix} 2x_2y_1 \\ -\sqrt{3}x_1y_2 - x_2y_2 \\ \sqrt{3}x_1y_3 - x_2y_3 \end{pmatrix}_{3(3')} + \begin{pmatrix} 2x_1y_1 \\ -x_1y_2 + \sqrt{3}x_2y_2 \\ -x_1y_3 - \sqrt{3}x_2y_3 \end{pmatrix}_{3'(3)}, \qquad (306)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3(3')} \times (y)_{1'} = \begin{pmatrix} x_1y \\ x_2y \\ x_3y \end{pmatrix}_{3'(3)}, \qquad (307)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1y_1 + x_2y_2)_1 + (x_1y_2 - x_2y_1)_{1'} + \begin{pmatrix} x_1y_2 + x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix}_2, \qquad (308)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \times (y)_{1'} = \begin{pmatrix} -x_2 y \\ x_1 y \end{pmatrix}_2, \tag{309}$$

$$(x)_{1'} \times (y)_{1'} = (xy)_1. \tag{310}$$

C Mass bounds of new particles

particle	mass	exp	particle	mass	exp
H^0(lightest even)	125.7	$125 - 126$[2]	$\chi_3^\pm$	1486	> 295 [16]
T_+	1882	> 560[16]	$\chi_w^\pm$	493	> 295 [16]
T_-	1178	> 560[16]	χ_1^0	199	> 46 [19]
G_+	3908		χ_2^0	493	> 62.4[19]
G_-	1737	(> 683)[16]	χ_3^0	1481	> 99.9[19]
$Q_{1,2}$	≥ 2532	> 1380[16]	χ_4^0	1487	> 116 [19]
$U_{1,2}^c$	≥ 2493	> 1380[16]	χ_5^0	2004	
$D_{1,2,3}^c$	≥ 2395	> 1380[16]	χ_6^0	2208	
$L_{1,2,3}$	≥ 1393	> 195[16]	$\chi_i^\pm$	119	$100 - 140$[15][16]
$E_{1,2,3}^c$	≥ 1490	> 195[16]	$\chi_{i,1}^0$	36.6	
$H_{1,2}^U(even, odd, \pm)$	1056	> 93.4[19]	$\chi_{i,2}^0$	120	> 116 [19]
$H_{1,2}^D(even, odd, \pm)$	821	> 93.4[19]	$\chi_{i,3}^0$	157	> 116 [19]
$S_{1,2}(even, odd)$	2052		g	2000	
$H_3(even, odd, \pm)$	2279	> 93.4[19]	λ_3^g	1000	> 1000[16]
$S_3(even)$	2102		Z'	2102	> 1520[7]

Table 7: Mass values of new particles calculated based on our assumption and corresponding experimental constraints in GeV units. The capital letters means bosons and the Greek characters and the small letter mean fermions. The equations which are used to calculate mass values, are Eq.(10),(71),(74),(75),(76),(77),(83),(85),(95),(96),(97),(98). Each equalities in "mass" column correspond to imposing the boundary conditions as $m_X^2 = 0 (X = Q, U_i^c, D_a^c, L_a, E_a^c)$ at $\mu = M_P$. We adopt the mass bound for stable stop as one for lighter G-Higgs (G_-) in bracket, under the assumption that G_- is lighter than g and G_+. We adopt the mass bound for CP-odd Higgs boson in supersymmetric model as ones for extra Higgs bosons ($H_{1,2}^U, H_{1,2}^D, H_3$).

The mass bound of the lightest chargino ($\chi_1^\pm$) is given by 3-lepton emission through EW direct process $\chi_1^\pm \chi_2^0 \to W^\pm Z \chi_1^0 \chi_1^0$. This neutralino χ_1^0 corresponds to $\chi_{i1,}^0$ in our model. Under the assumption that slepton decouples and LSP (χ_1^0) is massless, excluded region of chargino mass is given by $140 < M(\chi_1^\pm) < 295$GeV [16], or $M(\chi_1^\pm) < 330$GeV [17]. These constraints are not imposed on the chargino in the mass range

$$M(\chi_1^\pm) = M(\chi_2^0) < M(\chi_1^0) + m_Z. \tag{311}$$

In this case Z and following two lepton emissions are suppressed. Taking account of LEP bound $M(\chi_1^\pm) > 100$GeV [15], we consider the allowed region given by

$$100 < M(\chi_1^\pm) < 140 \quad \text{(GeV)}. \tag{312}$$

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